MAT 135A — Homework 6

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Solution of one selected problem

Problem 5: The joint density function is given by

$$f(x,y) = c(x^2 + xy/2), \quad 0 < x < 1,\ 0 < y < 2.$$

(a) Since $f(x,y)$ is a probability density function, we must have $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dxdy = 1$. We can evaluate

$$\begin{aligned}
\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dxdy &= c\int_0^1\int_0^2 (x^2 + xy/2)\,dydx \\
&= c\int_0^1 \left[x^2y + xy^2/4\Big|_0^2 \right]\,dx \\
&= c\int_0^1 (2x^2 + x)\,dx \\
&= c\left[\frac{2x^3}{3} + \frac{x^2}{2}\Bigg|_0^1 \right] \\
&= \frac{7c}{6}.
\end{aligned}$$

Therefore $c = \frac{6}{7}$.

(b) The marginal density is given by

$$\begin{aligned}
f_X(x) &= \int_{-\infty}^{\infty} f(x,y)\,dy \\
&= \frac{6}{7}\int_0^2 (x^2 + xy/2)\,dy \\
&= \frac{6}{7}\left[x^2y + xy^2/4\Big|_0^2 \right] \\
&= \frac{6}{7}(2x^2 + x), \quad 0 < x < 1.
\end{aligned}$$

(c)

$$\begin{aligned}
\mathbb{P}(X > Y) &= \int_{-\infty}^{\infty}\int_{-\infty}^{x} f(x,y)\,dydx \\
&= \frac{6}{7}\int_0^1\int_0^x (x^2 + xy/2)\,dydx \\
&= \frac{6}{7}\int_0^1 \left[x^2y + xy^2/4\Big|_0^x \right] \\
&= \frac{6}{7}\int_0^1 \frac{5x^3}{4}\,dx \\
&= \frac{6}{7}\left[\frac{5x^4}{16}\Bigg|_0^1 \right] = \frac{6}{7}\cdot\frac{5}{16} = \frac{15}{56}.
\end{aligned}$$