Solution of one selected problem

**Problem 7:** Since $X_i \sim \text{Exp}(\lambda)$, the pdf of each $X_i$ is given by

$$f_{X_i}(x_i) = \lambda e^{-\lambda x_i}, \quad 0 < x_i < \infty.$$ 

The probability distribution function of $X_i$ is given by

$$F_{X_i}(x) = \mathbb{P}(X_i \leq x) = \int_0^x \lambda e^{-\lambda x_i} \, dx_i = 1 - e^{-\lambda x}, \quad x > 0.$$ (1)

The probability distribution function of $H = \max\{X_1, \ldots, X_{10}\}$ is given by

$$F_H(x) = \mathbb{P}(H \leq x) = \mathbb{P}(X_1 \leq x, X_2 \leq x, \ldots, X_{10} \leq x) \quad (\text{since } \max\{X_1, \ldots, X_{10}\} \leq x \iff X_i \leq x \quad \forall i)$$

$$= \prod_{i=1}^{10} \mathbb{P}(X_i \leq x) \quad (\text{since } X_i \text{ s are independent of each other})$$

$$= (1 - e^{-\lambda x})^{10} \quad (\text{using (1)}).$$

Therefore the probability density function of $H$ is given by

$$f_H(x) = \frac{d}{dx} F_H(x) = 10\lambda e^{-\lambda x}(1 - e^{-\lambda x})^9, \quad x > 0,$$

observing the face that $X_i > 0 \Rightarrow H > 0$.

Similarly we can compute the probability distribution function of $L = \min\{X_1, \ldots, X_{10}\}$

$$F_L(x) = \mathbb{P}(L \leq x)$$

$$= 1 - \mathbb{P}(L > x)$$

$$= 1 - \mathbb{P}(X_1 > x, \ldots, X_{10} > x) \quad (\text{since } \min\{X_1, \ldots, X_{10}\} > x \iff X_i > x \quad \forall i)$$

$$= 1 - \prod_{i=1}^{10} \mathbb{P}(X_i > x) \quad (\text{since } X_i \text{ s are independent of each other})$$

$$= 1 - e^{-10\lambda x} \quad (\text{using (1)}).$$

Therefore the pdf of $L$ is

$$f_L(x) = \frac{d}{dx} F_L(x) = 10\lambda e^{-10\lambda x}, \quad x > 0.$$