MAT135a
Homework 2 (Due in class on January 28, 2015).

Reading: Please read pages 1-43 of the Gravner's notes

Problem 1.

You are walking on points in the planes with integer coordinates. Each time you can move either one unit up or one unit right; for example, from (1,3) you can move either to (2,3) or (1,4). You start at (1,1) and want to reach (4,5). (a) How many possible routes do you have? (b) You choose one of the routes at random. What is the probability that you visit (3,3) on your route?

Problem 2.

Three tours, A, B, and C, are offered to a group of 100 tourists. It turns out that 28 tourists sign for A, 26 for B, 16 for C, 12 for both A and B, 4 for both A and C, 6 for both B and C, and 2 for all three tours. (a) What is the probability that a randomly chosen tourist is taking none of these tours? (b) What is the probability that a randomly chosen tourist is taking exactly one of these tours? (c) What is the probability that two randomly chosen tourists are both taking at least one of these tours?

Problem 3.

An urn contains 5 red, 6 green, and 8 blue balls. Take three balls out at random one by one (a) without and (b) with replacement. In each case compute the probability that the balls are of (1) the same color and (2) three different colors.

Problem 4.

Assume a birthday is equally likely to be in any month of the year. In a group of 20 people, what is the probability that 4 months contain 2 birthdays each and 4 months contain 3 birthdays each?

Problem 5.

A man has 5 coins in his pocket. Two are double-headed, one is double- tailed, and two are normal. The coins cannot be distinguished unless one looks at them.

(a) The man shuts his eyes, chooses a coin at random, and tosses it. What is the probability that the lower face of the coin is heads?

(b) He opens his eyes and sees that the upper face of the coin is a head. What is the probability that the lower face is a head.

(c) He shuts his eyes again, picks up the same coin, and tosses it again. What is the probability that the lower face is a head?

(d) He opens his eyes and sees that the upper face is a head. What is the probability

that the lower face is a head?

Problem 6.

For events $A_1, A_2, \ldots A_n$ satisfying $P(A_1 \cap A_2 \cap \cdots \cap A_n) > 0$, prove that

$$P(A_1 \cap A_2 \cap \cdots \cap A_n) = P(A_1)P(A_2|A_1)P(A_3|A_1 \cap A_2) \cdots P(A_n|A_1 \cap A_2 \cap \cdots A_{n-1}).$$

Problem 7.

Students in a high school can choose to study Spanish or French as a foreign language. In a given academic year, 90 per cent of the pupils choose Spanish and the rest choose French. 30 per cent of the students who are learning Spanish are boys and 40 per cent who are studying French are also boys. If a student was randomly selected from this academic year, what is the probability that it would be a girl?