MAT135a
Homework 3 (Due in class on February 4, 2015).

Reading: Please read pages 24-53 of the Gravner's notes.

Problem 1.

You are dealt 13 cards from a shued deck of 52 cards. Compute the probability that (a) your hand lacks at least one suit, (b) you get the both Ace and King of at least one suit, (c) you get all four cards of at least one denomination (all Aces, or all Kings, or all Queens,..., or all Twos)

Problem 2.

Three boxes clearly labeled A, B, C contain three cards each. Here are the (known) values of the cards in each box: A: 1, 6, 8; B: 3, 5, 7; C: 2, 4, 9. Going first, Player 1 is free to select any of the three boxes, then chooses at random one of the cards from the box. Then Player 2 selects one of the remaining two boxes, and chooses at random a card from the box. The highest card wins. When you play the game, which of the two players would you prefer to be?

Problem 3.

There are two bags: A (contains 2 white and 4 red balls), B (1 white, 1 red). Select a ball at random from A, then put it into B. Then select a final ball at random from B. Compute (a) the probability that the final ball is white and (b) the probability that the transferred ball is white given that the final is white.

Problem 4.

Roll a die n times. Let A_{ij} be the event that the ith and jth rolls produce the same number. Show that the events $\{A_{ij} : 1 \leq i < j \leq n\}$ are pairwise independent but not independent.

Problem 5. A fair coin is tossed repeatedly. Show that the following two statements are equivalent:

(a) the outcomes of different tosses are independent,

(b) for any given finite sequence of heads and tails, the chance of this sequence occuring in the first m tosses is 2^{-m}, where m is the length of the sequence.

Problem 6.

You flip three fair coins. At least two are alike, and it is an even chance that the third is a head or a tail. Therefore $P(\textit{all three alike}) = 1/2$. Do you agree? Explain your reasoning.

Problem 7.

A biaed coin is tossed repeatedly. Each time there is a probability p of a head turining up. Let p_n be the probability that an even number of heads has occured after n tosses (zero is an even number). Show that $p_0 = 1$ and that

$$p_n = p(1 - p_{n-1}) + (1 - p)p_{n-1} \quad if n \geq 1.$$

Solve this difference equation.