MAT135a
Homework 4 (Due in class on February 13, 2015).

Reading: Please read pages 45-69 of the Gravner's notes.

Problem 1. Let X be the difference between the number of Heads and the number of Tails in n tosses of a fair coin. Compute the probability mass function of X.

Problem 2. A bag contains 5 red and 5 blue balls. Select two at random without replacement. If they are of the same color you win \$1.10 otherwise you lose \$1. Let X be your winnings. Compute EX and $Var(X)$.

Problem 3. A multiple choice exam has 5 questions, with three choices for each question. The passing score is at least four correct answers. (a) What is the probability that a student who answers each question at random passes the test? (b) Assuming a class has 50 students, and all answer each question at random, what is the expected number of students that pass the tesst?

Problem 4. Assume the suicide rate is 1 per 100,000 people per month, and a city has 400,000 inhabitants. (a) Find the probability that there will be 8 or more suicides next month in this city. (b) Find the probability that next year there will be at least two months with 8 or more suicides. (c) Counting the next month as month 1, what is the probability that the first month to have 8 or more suicides will be month i ?

Problem 5.

Is it generally true that $E(1/X) = 1/E(X)$? Is it ever true that $E(1/X) = 1/E(X)$?

Problem 6. Let $X_i,\ 1 \le i \le n$, be independent random variables which are symmetric about 0; that is X_i and $-X_i$ have the same distributions. Show that, for all x, $P(S_n \ge x) = P(S_n \le -x)$ where $S_n = \sum_{i=1}^{n} X_i$. Is the conclusion necessarily true without the assumption of independence?

Problem 7.

Let X have probability mass function

$$f(x) = \frac{1}{x(x+1)}, \quad x = 1, 2, 3, \ldots$$

(X takes values in the set of positive integers). For what values of $a \in R$ we have

$$E(X^a) < \infty?$$