MAT135a

Homework 5 (Due in class on February 20, 2015).

Reading: Please read pages 58-78 of the Gravner's notes.

Problem 1. Assume that weekly sales of diesel fuel at a gas station are X tons, where X is a random variable with distribution function

$$f(x) = c(1-x)^4, \quad 0 < x < 1$$

(the density is zero outside the interval $(0,1)$).

(a) Compute c.

(b) Compute EX.

(c) The current tank (of capacity 1) will be emptied and out for repairs next week; to save money, the station wants to rent a tank for the week with capacity just large enough that its supply will be exhausted with probability 0.01. What is the capacity of the tank?

Problem 2.

If X is an Exponential(1) random variable, compute the density of

(a) $Y = \log X$, and (b) $Z = (\log X)^2$.

Problem 3.

Assume A is uniform in $[0,5]$. Compute the probability that the equation

$$4x^2 + 4Ax + A + 2 = 0$$

has two real roots.

Problem 4.

January snowfall, in inches, in Truckee, California has expectation 50 and standard deviation 35. (These are close to true numbers, from the ranger station website.) Assume normal distribution and year to year independence. What is the probability that, starting from the next January, it will take at least 11 years to get January snowfall over 100 inches?

Problem 5.

Let X and Y be independent random variables with common distribution function F and density function f. Show that $V = \max[X, Y]$ has distribution function $P(V \le x) = F(x)^2$ and density function $f_V(x) = 2f(x)F(x)$, $x \in R$. Find the density function of $U = \min[X, Y]$.