MAT135a
Homework 6 (Due in class on February 27, 2015).

Reading: Please read pages 58-87 of the Gravner’s notes.

Problem 1.

Mr. Smith is selling his house, and has decided to accept the first offer exceeding $K$ US Dollars. Assuming that offers are independent random variables with common distribution function $F$, find the expected numbers of offers received before he sells the house.

Problem 2.

Let $X$ have the uniform distribution on $[0, 1]$. For what function $g$ does $Y = g(X)$ have the exponential distribution with parameter 1?

Problem 3. [Log-normal distribution]

Let $Y = e^X$, where $X$ has the standard normal distribution $N(0,1)$. Find the density function of $Y$.

Problem 4.

A fair die is rolled twice. Compute the joint p.m.f. of $X$ and $Y$, where $X$ is the first number rolled and $Y$ is the largest of the two numbers rolled.

Problem 5.

Joint density of $(X, Y)$ is given by

$$f(x, y) = c(x^2 + xy/2), \quad 0 < x < 1, \quad 0 < y < 2.$$  

(a) Compute $c$.

(b) Compute the density of $X$.

(c) Compute $P(X > Y)$.

Problem 6.

Mr. Smith arrives at a location at a time uniformly distributed between 12:15 and 12:45, while Mrs. Smith independently arrives at the same location at a time uniformly distributed between 12 and 1 (all times p.m.). (a) Compute the probability that that the first person to arrive waits no longer than 5 minutes. (b) Compute the probability that Mr. Smith arrives first.