

MAT135a
Homework 6 (Due in class on March 6, 2015).

Reading: Please read pages 72-104 of the Gravner's notes.

Problem 1.

Joint density of (X, Y) is given by

$$f_1(x, y) = x \exp(x + y), \quad x, y > 0,$$

(and zero otherwise).

- a) Are X and Y independent?
- (b) Repeat with joint density

$$f_2(x, y) = 2, \quad 0 < x < y < 1,$$

(and zero otherwise).

Problem 2.

Assume that X_1, \dots, X_{10} are independent Exponential(λ) random variables. Compute the densities of $H = \max(X_1, \dots, X_{10})$, and $L = \min(X_1, \dots, X_{10})$.

Problem 3.

A coin has probability p of Heads. Toss this coin n times, and let X be the number of tosses, from toss 2 on, that have different outcome than the previous toss. Compute EX .

Problems 4.

The annual rainfall figures in Bandrika are independent identically distributed continuous random variables $\{X_n : n \geq 1\}$. Find the probability that

- (a) $X_1 < X_2 < X_3 < X_4$.
- (b) $X_1 > X_2 < X_3 < X_4$.

Problems 5.

Find the density function of $Z = X + Y$ when X and Y have joint probability density function

$$\frac{1}{2}(x + y)e^{-(x+y)}, \quad x, y \geq 0.$$