MAT135a
Homework 6 (Due in class on March 6, 2015).

Reading: Please read pages 72-104 of the Gravner’s notes.

Problem 1.

Joint density of \((X,Y)\) is given by

\[
f_1(x, y) = x \exp(x + y), \quad x, y > 0,
\]
(and zero otherwise).

(a) Are \(X\) and \(Y\) independent?

(b) Repeat with joint density

\[
f_2(x, y) = 2, \quad 0 < x < y < 1,
\]
(and zero otherwise).

Problem 2.

Assume that \(X_1, \ldots, X_{10}\) are independent Exponential(\(\lambda\)) random variables. Compute the densities of \(H = \max(X_1, \ldots, X_{10})\), and \(L = \min(X_1, \ldots, X_{10})\).

Problem 3.

A coin has probability \(p\) of Heads. Toss this coin \(n\) times, and let \(X\) be the number of tosses, from toss 2 on, that have different outcome than the previous toss. Compute \(EX\).

Problems 4.

The annual rainfall figures in Bandrika are independent identically distributed continuous random variables \(\{X_n : n \geq 1\}\). Find the probability that

(a) \(X_1 < X_2 < X_3 < X_4\).

(b) \(X_1 > X_2 < X_3 < X_4\).

Problems 5.

Find the density function of \(Z = X + Y\) when \(X\) and \(Y\) have joint probability density function

\[
\frac{1}{2}(x + y)e^{-(x+y)}, \quad x, y \geq 0.
\]