MAT202
Final Exam (Due March 21, 2019 by 1pm).

Problem 1.

Let A be a multiplication operator $Af = gf$ on $L^2[0,1]$ where $g(x)$ is a real-valued continuous function. Prove that A is self-adjoint and find its spectrum.

Problem 2.

Let $K(x,y)$ be a continuous function on the unit square $0 \leq x, y \leq 1$. Prove that the operator A on $C[0,1]$ defined as

$$Af(x) = \int_0^1 K(x,y)f(y)dy$$

is compact.

Problem 3.

Let $B : X \to X$ be a linear operator on a Banach space X such that $Ran(A)$ is closed and there exists $C > 0$ such that $||Ax|| \geq C||x||$ for all x. Prove that A is closed.

Problem 4.

Prove that an idenitity operator is not compact in infinite-dimensional normed space.

Problem 5.

Is it possible for a compact operator A to satisfy $p(A) = 0$ where $p(x) = c_0 + c_1 \times x + \ldots + c_n x^n$ is a non-zero polynomial? Hint: the answer depends on whether c_0 is zero or not. Consider both cases.

Problem 6.

Let $A \geq 0$. Prove that $(A - \lambda)^{-1}$ exists for all $\lambda < 0$.