MAT 21A First Midterm Exam

Last Name (PRINT): ________________________________

First Name (PRINT): ________________________________

Student ID #: ________________________________

Section: ________________________________

Instructions:

1. Do not open your test until you are told to begin.
2. Use a pen to print your name in the spaces above.
3. No notes, books, calculators, or any other electronic devices allowed.
4. Show all your work. Unsupported answers will receive NO CREDIT.
5. You are expected to do your own work.

#1	#2	#3	#4	#5	#6	#7	#8	#9	T

1. Evaluate

$$\lim_{x \to 0} (2 + \sin x)(x^3 - 8)$$

Solution

$$\lim_{x \to 0} (2 + \sin x)(x^3 - 8) = \lim_{x \to 0} (2 + \sin x) \lim_{x \to 0} (x^3 - 8) = 2(-8) = -16$$

2. Evaluate

$$\lim_{x \to 1^-} \sqrt{1 - x^2}.$$

Solution

$$\lim_{x \to 1^-} \sqrt{1 - x^2} = \sqrt{1 - (1^-)^2} = 0$$

3. Evaluate

$$\lim_{x\to 1} \frac{x^2 + x - 2}{x^2 - x}$$

Solution

$$\lim_{x\to 1} \frac{x^2 + x - 2}{x^2 - x} = \lim_{x\to 1} \frac{(x+2)(x-1)}{x(x-1)} \lim_{x\to 1} \frac{x+2}{x} = 3$$

4. If

$$3 - 3x^2 \leq g(x) \leq 3\cos x$$

for all x, find $\lim_{x \to 0} g(x)$.

Solution By the Sandwhich theorem:

$$3 = \lim_{x \to 0} 3 - 3x^2 \leq \lim_{x \to 0} g(x) \leq \lim_{x \to 0} 3\cos x = 3$$

So we have $\lim_{x \to 0} g(x) = 3$

5. Prove that

$$\lim_{x \to 2} 2x + 6 = 10$$

using the $\epsilon - \delta$ definition of the limit.

Solution Let $\epsilon > 0$ be given. The goal is to find $\delta > 0$ such that $|x - 2| < \delta$ guarantees:

$$|2x + 6 - 10| = |2x - 4| < \epsilon$$

In fact we have:

$$|2x - 4| = 2|x - 2| < 2\delta$$

So making $\delta = \epsilon/2$ suffices. Suppose ϵ is given and $|x - 2| < \delta$ then:

$$|2x + 6 - 10| = |2x - 4| < 2\delta = \epsilon$$

6. At what points is the function $f(x) = \frac{x+1}{x^2-5x+6}$ continuous?

Solution Polynomials are continuous functions. A quotient of two polynomials then only fails to be continuous whenever the denominator vanishes.

$$(x^2 - 5x + 6) = (x - 2)(x - 3)$$

So the function is continuous everywhere except at $x = 2$ and $x = 3$

7. Find the limit

$$\lim_{x\to\infty} \sqrt{x^2+9} - \sqrt{x^2+1}.$$

Solution

$$\lim_{x\to\infty} \sqrt{x^2+9} - \sqrt{x^2+1} = \lim_{x\to\infty} \frac{(\sqrt{x^2+9} - \sqrt{x^2+1})(\sqrt{x^2+9} + \sqrt{x^2+1})}{(\sqrt{x^2+9} + \sqrt{x^2+1})}$$

$$= \lim_{x\to\infty} \frac{x^2+9-x^2-1}{(\sqrt{x^2+9} + \sqrt{x^2+1})} = \lim_{x\to\infty} \frac{8}{(\sqrt{x^2+9} + \sqrt{x^2+1})} = 0$$

8. Find the derivative of $f(x) = x^2$ at $x = 2$ using the definition of the derivative.

Solution By the definition of the derivative:

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(2+h)^2 - 2^2}{h} = \lim_{h \to 0} = \frac{4 + h^2 + 4h - 4}{h} = \lim_{h \to 0} 4 + h = 4$$

9. Find the slope of the tangent line to the curve $y = \sqrt{x}$ at the point $x = 4$.

Solution The slope of the tangent line is given by the derivative. As in the previous exercise we can compute this with the limit definition:

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{(4+h)} - \sqrt{4}}{h} = \lim_{h\to 0} \frac{(\sqrt{(4+h)} - \sqrt{4})(\sqrt{4+h} + \sqrt{4})}{h(\sqrt{4+h} + \sqrt{4})}$$

$$\lim_{h\to 0} \frac{h}{h(\sqrt{4+h} + \sqrt{4})} = \lim_{h\to 0} \frac{1}{(\sqrt{4+h} + \sqrt{4})} = \frac{1}{2\sqrt{4}} = \frac{1}{4}$$