5. It takes a force of 21,714 lb. to compress a coil spring assembly on a New York City Transit Authority subway car from its free height of 8 in. to its fully compressed height of 5 in.

a) What is the assembly's force constant (a.k.a. spring constant)?

b) How much work does it take to compress the assembly the first half inch? the second half inch?

\[ F = kx = k(8 - 5) = 3k \]

\[ 21,714 = 3k \implies k = \frac{21,714}{3} = 7,238 \text{ lb/in} \]

b) \[
W_1 = \int_0^{\frac{1}{2}} F(x) \, dx \quad \text{first half inch}
\]

\[
W_2 = \int_{\frac{1}{2}}^1 F(x) \, dx \quad \text{second half inch}
\]

In our case, \( F(x) = kx \). Thus

\[
W_1 = \int_0^{\frac{1}{2}} kx \, dx = k \left[ \frac{x^2}{2} \right]_0^{\frac{1}{2}} = \frac{k}{4} = \frac{7,238}{8} = 904 \text{ lb-in}
\]

\[
W_2 = \int_{\frac{1}{2}}^1 kx \, dx = k \left[ \frac{x^2}{2} \right]_{\frac{1}{2}}^1 = \frac{k}{2} - \frac{k}{8} = \frac{3}{8}k = \frac{3}{8} \times 7,238 = 2,184 \text{ lb-in}
\]
6. Find the center of mass of a thin plate of constant density \( \delta \) covering the region bounded by the \( y \)-axis and the curve \( x = y - y^3, \ 0 \leq y \leq 1 \).

Using the formulas on p. 404,

\[
A(D) = \int_0^1 (y - y^3) \, dy = \left[ \frac{y^2}{2} - \frac{y^4}{4} \right]_0^1 = \frac{1}{4}
\]

\[
\bar{x} = \frac{\int_0^1 y(y - y^3) \, dy}{A(D)} = \frac{4}{A(D)} \int_0^1 y(y - y^3) \, dy = 4 \int_0^1 (y^2 - y^4) \, dy
\]

\[
= 4 \left( \frac{y^3}{3} - \frac{y^5}{5} \right)_0^1 = 4 \left( \frac{1}{3} - \frac{1}{5} \right) = \frac{8}{15}
\]

\[
\bar{y} = \frac{\int_0^1 \frac{1}{2} (y - y^3)^2 \, dy}{A(D)} = \frac{2}{A(D)} \int_0^1 (y - y^3)^2 \, dy
\]

\[
= 2 \int_0^1 (y^2 - 2y^4 + y^6) \, dy = 2 \left( \frac{y^3}{3} - \frac{2y^5}{5} + \frac{y^7}{7} \right)_0^1
\]

\[
= 2 \left( \frac{1}{3} - \frac{2}{5} + \frac{1}{7} \right) = 2 \left( \frac{35}{105} - \frac{42}{105} + \frac{15}{105} \right) = \frac{16}{105}
\]
7. Solve the differential equation 

\[ 2\sqrt{xy} \frac{dy}{dx} = 1, \quad x, y > 0. \]

This is a Separa\textbf{ble} Differential Equation (see p\textsuperscript{1} 428 - 430). We write:

\[ \frac{dy}{dx} = \frac{1}{2 \sqrt{xy}} \]

\[ \sqrt{y} \ dy = \frac{1}{2} \frac{dx}{\sqrt{x}} \]

\[ \int \sqrt{y} \ dy = \frac{1}{2} \int \frac{dx}{\sqrt{x}} \]

\[ \frac{2}{3} y^{\frac{3}{2}} = x^{\frac{1}{2}} + C \]

\[ y^{\frac{3}{2}} = \frac{3}{2} x^{\frac{1}{2}} + C_1 \]

\[ y = \left( \frac{3}{2} x^{\frac{1}{2}} + C_1 \right)^{\frac{2}{3}}, \quad C_1 > 0. \]
8. Suppose the amount of oil pumped from one of the canyon wells in Whittier, California decreases at the continuous rate of 10% per year. When will the well's output fall to one-fifth of its present value?

Look at the Exponential Change section (p. 428).

We have

\[ y(t) = y(0) e^{kt} \],

where

\[ k = \ln(0.9) \], so \[ y(t) = y(0) (0.9)^t \]

\[ y(t) = \frac{4}{5} y(0) \Rightarrow \]

\[ \frac{4}{5} = (0.9)^t = e^{\ln 0.9 \cdot t} \]

\[ \Rightarrow \ln 0.2 = t \ln 0.9 \]

\[ \Rightarrow t = \frac{\ln 0.2}{\ln 0.9} = \frac{\ln 5}{\ln 10 - \ln 9} \]