1. Which formula is not equivalent to the other two?

a.
$$\sum_{n=3}^{6} \frac{(-1)^{n-1}}{n-1}$$

b.
$$\sum_{n=1}^{4} \frac{(-1)^{n}}{n+1}$$

c.
$$\sum_{n=0}^{3} \frac{(-1)^{n}}{n+2}.$$

Explain your answer.

b) is not equivalent to a) and c)

since

$$a) = \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5},$$

$$b) = -\frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5}, \text{ and}$$

$$c) = \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5}$$

2. Suppose that f and g are integrable and that

$$\int_4^9 f(x)dx = 5$$

and

$$\int_4^9 g(x)dx = -3.$$

Find

$$\int_2^3 x(f(x^2) - 2g(x^2))dx.$$

$$\int_2^3 x \left(f(x^2) - 2g(x^2)\right) dx$$

$$= \int_2^3 \left(f(x^2) - 2g(x^2)\right) \frac{1}{2}\, d(x^2)$$

$$= \int_{2^2}^{3^2} \left(f(u) - 2g(u)\right) \frac{1}{2}\, du$$

$$= \frac{1}{2} \int_4^9 f(u)\, du - \int_4^9 g(u)\, du$$

$$= \frac{5}{2} + 3 = \frac{11}{2}$$

3. Could it be that

$$\int_0^{\pi/2} 3\sin(x^3)dx = 2\pi \ ?$$

Explain your answer.

$\triangleright$ $3\sin(x^3) \leq 3$ since $\sin(y) \leq 1$

Thus,

$$\int_0^{\frac{\pi}{2}} 3\sin(x^3)\,dx \leq \int_0^{\frac{\pi}{2}} 3\,dx = \frac{3\pi}{2} < 2\pi.$$

Therefore, the integral cannot equal 2π.

4. Find $f(2012)$ if

$$\int_1^x f(t)dt = (x-1)\cos(\pi x).$$

Let $F(x) = \int_1^x f(t)\,dt$.

Then by the Fundamental Theorem of Calculus,

$$F'(x) = f(x).$$

Thus $f(x) = \frac{d}{dx}\left[(x-1)\cos \pi x\right]$

$= \cos \pi x + (x-1)(-\pi \sin \pi x)$

Now, $f(2012) = \cos(\pi \times 2012)$

$- 2011\,\pi \sin(2012\pi)$

$= 1 - 2011 \times \pi \times 0 = 1.$