

1. Which formula is not equivalent to the other two?

a.

$$\sum_{n=3}^6 \frac{(-1)^{n-1}}{n-1}$$

b.

$$\sum_{n=1}^4 \frac{(-1)^n}{n+1}$$

c.

$$\sum_{n=0}^3 \frac{(-1)^n}{n+2}$$

Explain your answer.

b) is not equivalent to a) and c)

since

$$a) = \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5},$$

$$b) = -\frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5}, \text{ and}$$

$$c) = \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5}$$

2. Suppose that f and g are integrable and that

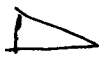
$$\int_4^9 f(x) dx = 5$$

and

$$\int_4^9 g(x) dx = -3.$$

Find

$$\int_2^3 x(f(x^2) - 2g(x^2)) dx.$$



$$\int_2^3 x (f(x^2) - 2g(x^2)) dx$$

$$= \int_2^3 (f(x^2) - 2g(x^2)) \frac{1}{2} d(x^2)$$

$$= \int_{2^2}^{3^2} (f(u) - 2g(u)) \frac{1}{2} du$$

$$= \frac{1}{2} \int_4^9 f(u) du - \int_4^9 g(u) du$$

$$= \frac{5}{2} + 3 = \frac{11}{2}$$

3. Could it be that

$$\int_0^{\pi/2} 3 \sin(x^3) dx = 2\pi?$$

Explain your answer.

$$\triangleright \quad 3 \sin(x^3) \leq 3 \quad \text{since} \quad \sin(y) \leq 1$$

Thus,

$$\int_0^{\frac{\pi}{2}} 3 \sin(x^3) dx \leq \int_0^{\frac{\pi}{2}} 3 dx = \frac{3\pi}{2} < 2\pi.$$

Therefore, the integral cannot equal 2π .

4. Find $f(2012)$ if

$$\int_1^x f(t) dt = (x-1) \cos(\pi x).$$



$$\text{Let } F(x) = \int_1^x f(t) dt.$$

Then by the Fundamental Theorem
of Calculus,

$$F'(x) = f(x).$$

$$\text{Thus } f(x) = \frac{d}{dx} [(x-1) \cos \pi x]$$

$$= \cos \pi x + (x-1)(-\pi \sin \pi x)$$

$$\text{Now, } f(2012) = \cos(\pi \times 2012)$$

$$- 2011 \pi \sin(2012 \pi)$$

$$= 1 - 2011 \times \pi \times 0 = 1.$$