1. Scientists who do carbon-14 dating use a figure of 5700 years for its half-life. Find the age of a sample in which 30 percent of the radioactive nuclei originally present have decayed.

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half-life $\quad T = \dfrac{\ln 2}{k} \Rightarrow k = \dfrac{\ln 2}{T} = \dfrac{\ln 2}{5,700}$

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Find $t$ so that

$$e^{-kt} = 0.7$$

$$-kt = \ln 0.7$$

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$$t = \frac{-\ln 0.7}{k}$$

$$= \frac{-\ln 0.7}{\ln 2} \cdot 5,700$$

2. A spring has a natural length of 2 m. A force of 30 N holds the spring stretched to a total length of 3 m. a) Find the force constant. b) How much work will it take to stretch the spring 2 m beyond its natural length?

a) $F = k x$

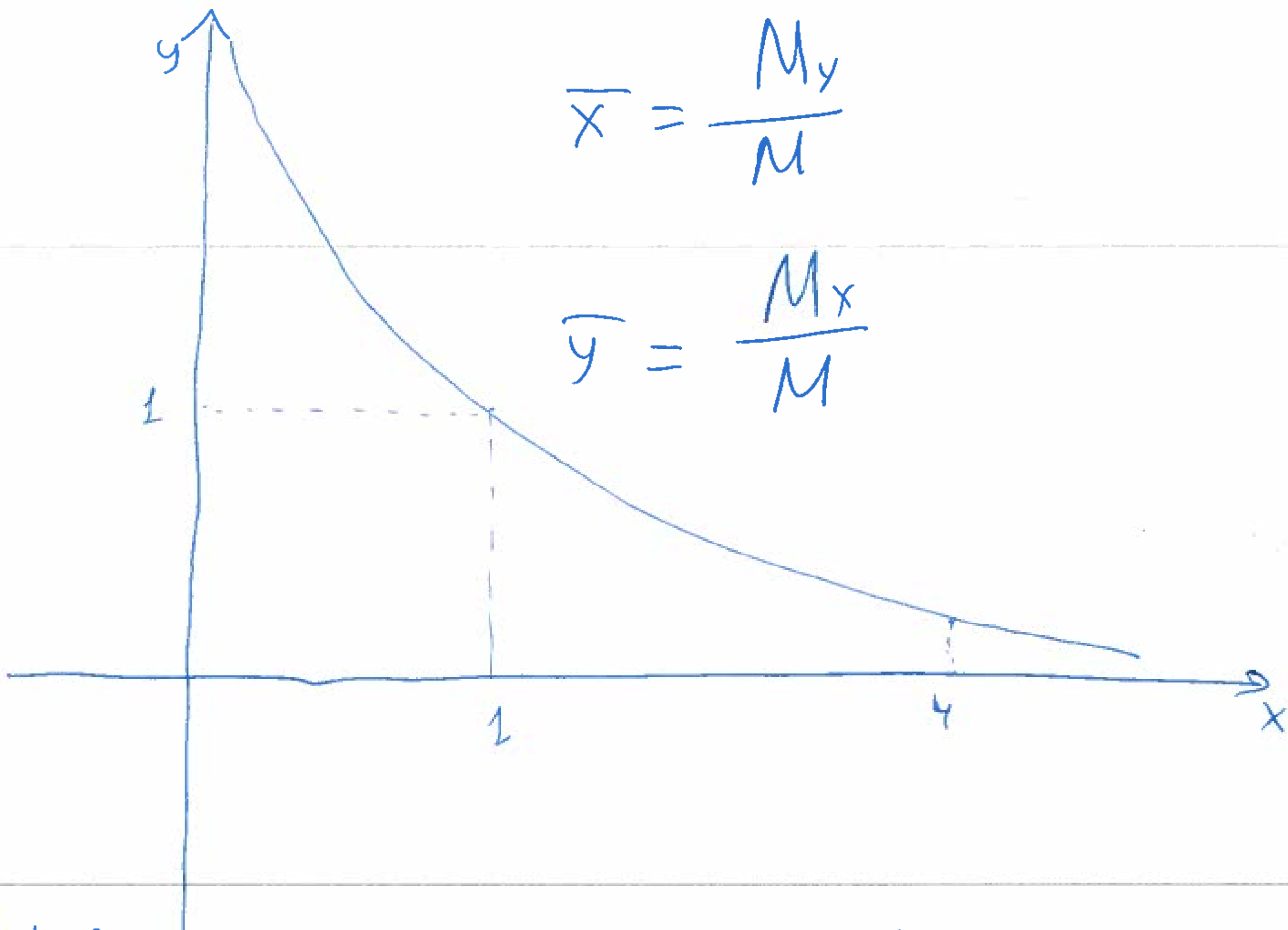
$30 = k(3-2)$

$\Rightarrow k = 30$

b)

$F = k x = 30 \cdot 2 = 60 \, N$

3. Find the center of mass of a thin plate covering the region between the $x$-axis and the curve $y = 1/x^2$, $1 \le x \le 4$, if the plate's density at the point $(x, y)$ is $\delta(x) = x^2$.




$$\bar{x} = \frac{M_y}{M}$$

$$\bar{y} = \frac{M_x}{M}$$

total mass

$$M = \int_1^4 \delta(x) \cdot \frac{1}{x^2}\, dx = \int_1^4 x^2 \cdot \frac{1}{x^2}\, dx = 3$$

$$M_y = \int_1^4 x\, \delta(x) \cdot \frac{1}{x^2}\, dx = \int_1^4 x\, dx = \frac{x^2}{2}\Big|_1^4 = \frac{15}{2}$$

$$M_x = \int_1^4 \frac{1}{2x^2} \cdot \delta(x) \cdot \frac{1}{x^2}\, dx = \int_1^4 \frac{1}{2x^2}\, dx = -\frac{1}{2x}\Big|_1^4$$

$$= \frac{1}{2} - \frac{1}{8} = \frac{3}{8}$$

4. The graph of the equation $x^{2/3} + y^{2/3} = 1$ is one of a family of curves called *astroids* because of their starlike appearance. Find the length of this particular astroid by finding the length of half of the first-quadrant portion, $y = (1 - x^{2/3})^{3/2}$, $\sqrt{2}/4 \le x \le 1$, and multiplying by 8.

$$\ell = 8 \int_{\frac{\sqrt{2}}{4}}^{1} \sqrt{1 + (y'(x))^2}\, dx$$

Now,

$$y'(x) = \frac{3}{2}\left(1 - x^{\frac{2}{3}}\right)^{\frac{1}{2}} \cdot \left(-\frac{2}{3} x^{-\frac{1}{3}}\right)$$

$$= -x^{-\frac{1}{3}}\left(1 - x^{\frac{2}{3}}\right)^{\frac{1}{2}}$$

$$= -\left(-1 + x^{-\frac{2}{3}}\right)^{\frac{1}{2}}$$

Thus,

$$\sqrt{1 + (y')^2} = \sqrt{1 - 1 + x^{-\frac{2}{3}}}$$

$$= x^{-\frac{1}{3}}, \quad \text{and}$$

$$\ell = 8 \int_{\frac{\sqrt{2}}{4}}^{1} x^{-\frac{1}{3}} = 8 \cdot \frac{3}{2} x^{\frac{2}{3}} \Big]_{\frac{\sqrt{2}}{4}}^{1} =$$

5. Evaluate the integral $\int x^2 e^x dx$.

$$\int x^2 e^x dx$$

$$= \int (e^x)' x^2 dx = e^x x^2 - \int (e^x)(x^2)' dx$$

$$= e^x \cdot x^2 - 2 \int e^x \cdot x \, dx$$

$$= e^x \cdot x^2 - 2 \int (e^x)' \cdot x \, dx$$

$$= e^x \cdot x^2 - 2 \left[ e^x \cdot x - \int e^x \cdot (x)' dx \right]$$

$$= e^x \cdot x^2 - 2xe^x + \int e^x dx$$

$$= e^x \cdot x^2 - 2xe^x + e^x + C$$