1. Find indefinite integral in a) and definite integral in b).

a)

$$\int t^{-2}dt,$$

b)

$$\int_0^1 (10e^{-x} + \cos x)dx.$$

a) $\int t^{-2} dt = -\frac{1}{t} + C$

b) $\int_0^1 (10e^{-x} + \cos x)\, dx$

$= 10\int_0^1 e^{-x} dx + \int_0^1 \cos x \, dx$

(by Fundamental Thm. of Calculus)

$= 10 \cdot (-e^{-x})\Big|_0^1 + \sin x \Big|_0^1$

$= 10\left(-\frac{1}{e} + 1\right) + \sin 1 - \sin 0$

$= 10(1 - e^{-1}) + \sin 1$

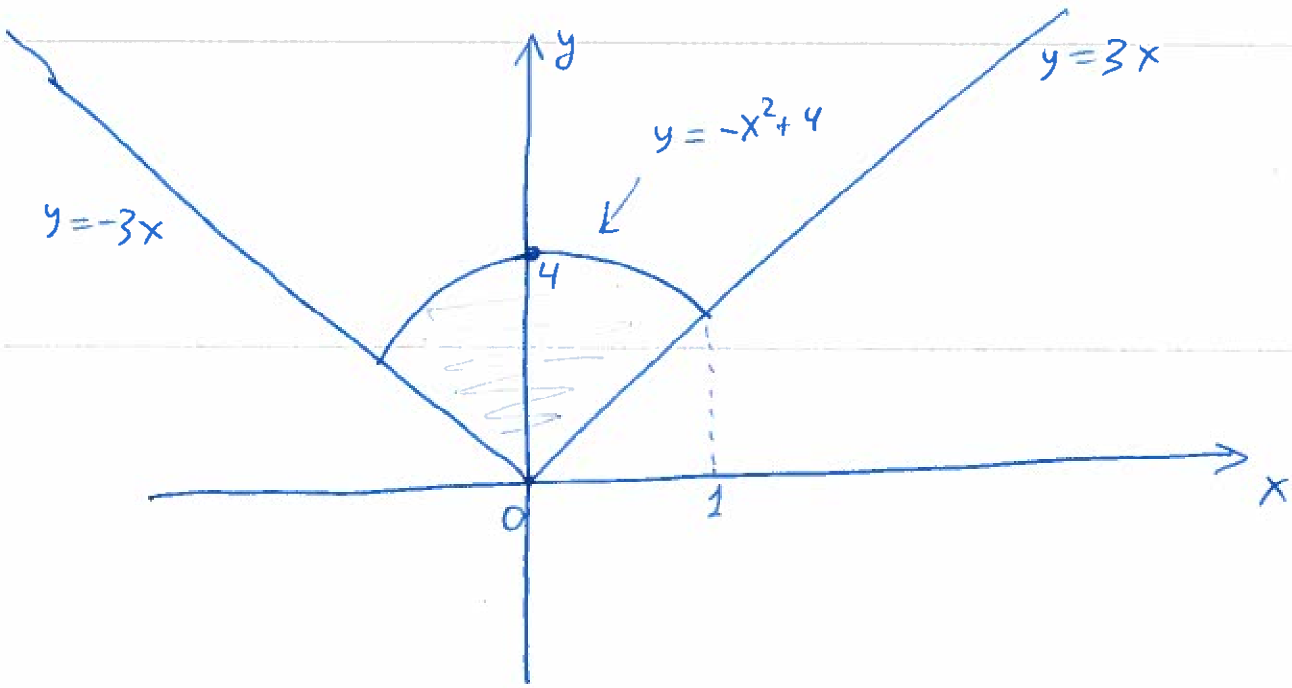
2. Suppose that $\int_1^x f(t)dt = x^2 - 2x + 1$. Find $f(x)$.

By the Fundamental Thm of Calculus,

$$\frac{d}{dx} \int_1^x f(t)\, dt = f(x).$$

Thus, $f(x) = (x^2 - 2x + 1)' = 2x - 2$

3. Find the area of the region enclosed by the curves $y = -x^2 + 4$ and the line $y = 3|x|$.



$$-x^2+4 = 3x \quad , \quad x>0$$

$$x^2+3x-4=0 \quad , \quad x>0$$

$$(x+4)(x-1)=0 \quad , \quad x>0 \quad \Rightarrow \quad x=1$$

$$\text{Area} = 2\int_0^1 (-x^2+4) - 3x \; dx$$

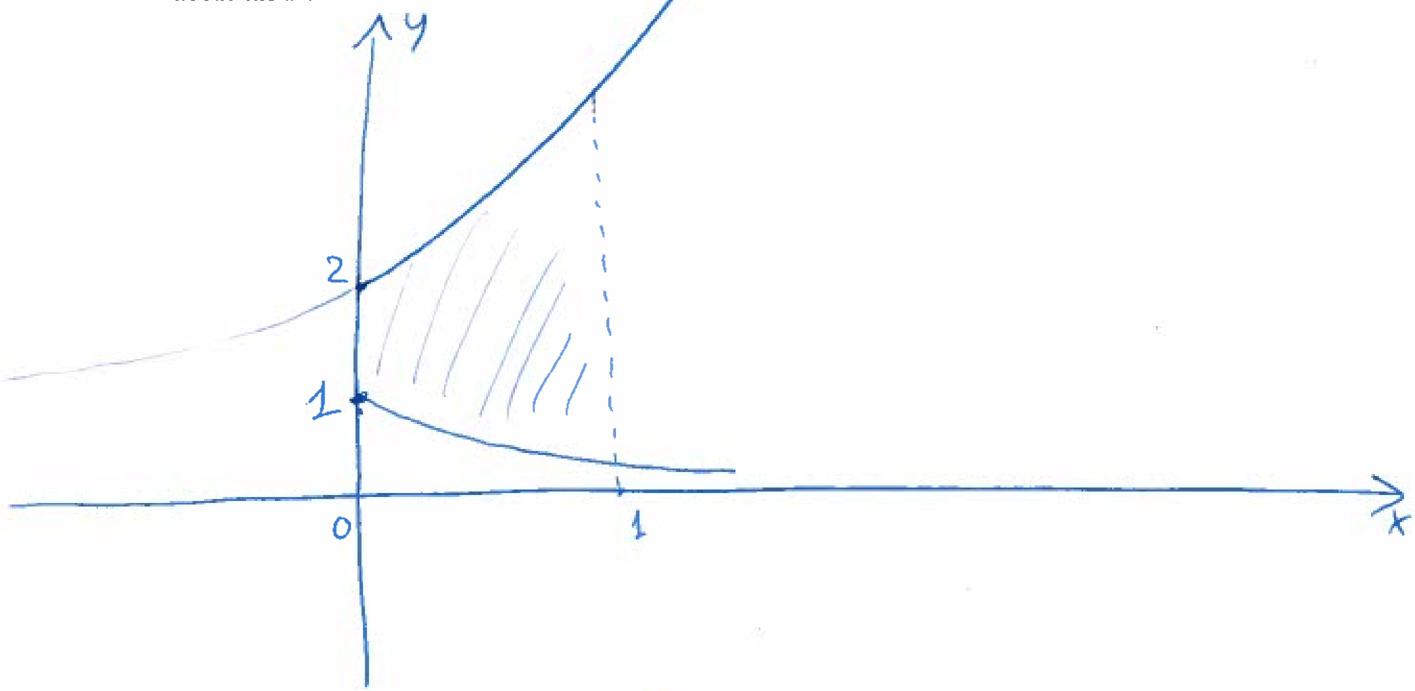
$$= 2\int_0^1 -x^2\,dx + 2\int_0^1 4\,dx + 2\int_0^1 -3x\,dx = -\frac{2}{3}x^3\Big|_0^1$$

$$+ 8 - 3x^2\Big|_0^1 = -\frac{2}{3} + 8 - 3 =$$

4. Find the volume of the solid generated by revolving the region bounded by the lines and curves

$$y = 2e^x, \quad y = e^{-x}, \quad x = 0, \quad x = 1$$

about the x-axis.



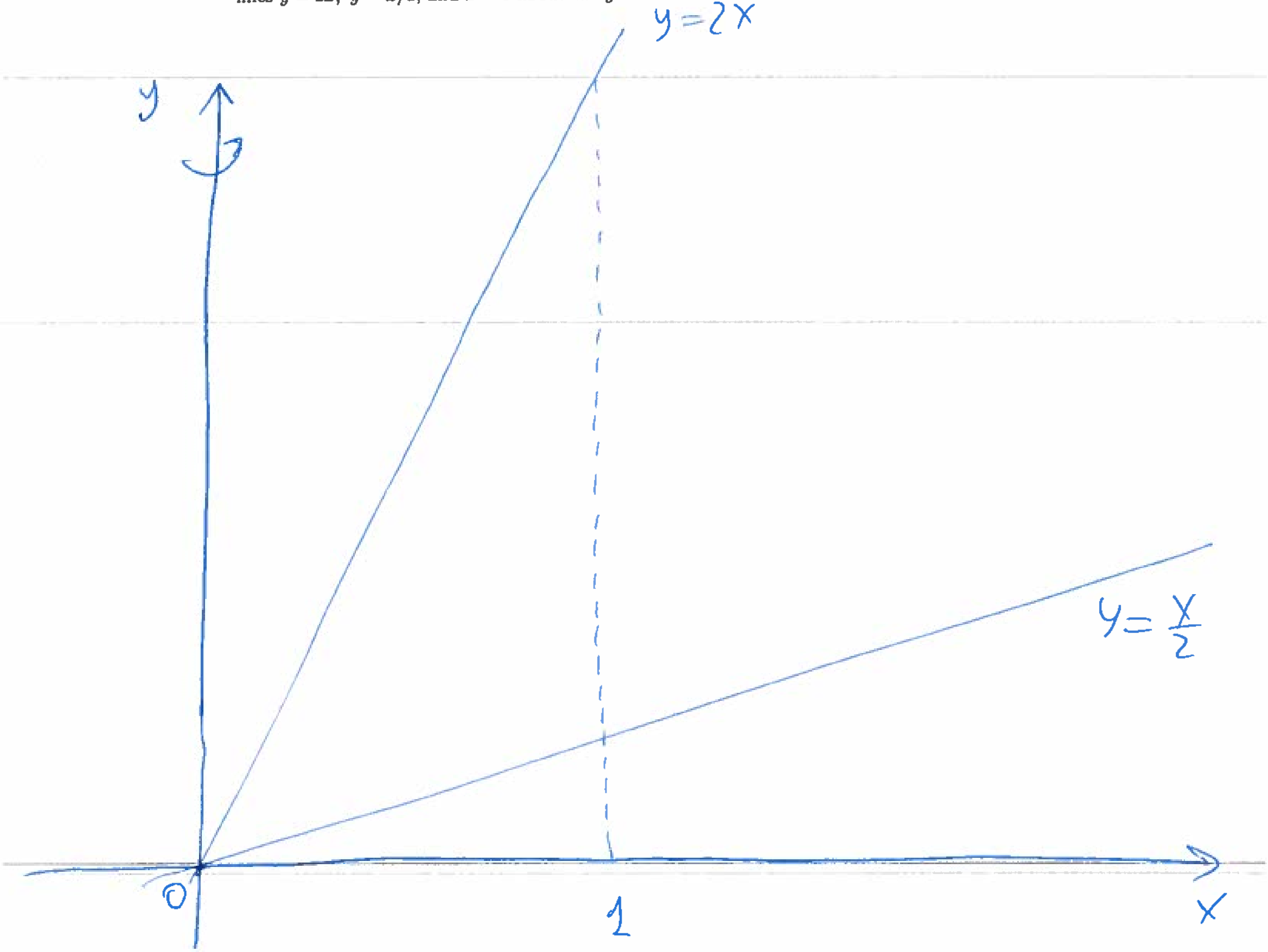
Method of washers

$$\text{Volume} = \int_0^1 \pi (2e^x)^2 - \pi (e^{-x})^2 \, dx$$

$$= 4\pi \int_0^1 e^{2x} \, dx - \pi \int_0^1 e^{-2x} \, dx$$

$$= 2\pi \, e^{2x}\Big|_0^1 + \frac{\pi}{2} \, e^{-2x}\Big|_0^1 = 2\pi e^2 - 2\pi + \frac{\pi}{2} e^{-2} - \frac{\pi}{2}$$

5. Use the shell method to find the volume of the solid generated by revolving the region bounded by the lines $y = 2x$, $y = x/2$, and $x = 1$ about the y-axis.



$$V = 2\pi \int_0^1 R(x)\ H(x)\, dx = 2\pi \int_0^1 x \cdot \left(2x - \frac{x}{2}\right) dx$$

$$= 2\pi \int_0^1 \frac{3}{2} x^2\, dx = 2\pi\ \frac{1}{2} x^3 \Big|_0^1 = 2\pi\left[\frac{1}{2} - 0\right] = \pi$$

6. a) Show that the value of $\int_0^8 \sqrt{x+1}dx$ lies between 8 and 24 without actually computing the integral.

b) Compute the value of $\int_0^8 \sqrt{x+1}dx$.

a) By monotonicity

$$\sqrt{0+1} \le \sqrt{x+1} \le \sqrt{8+1} \quad \text{for } 0 \le x \le 8$$

so

$$1 \le \sqrt{x+1} \le 3 \quad \text{for } 0 \le x \le 8$$

Thus $8 \cdot 1 \le \int_0^8 \sqrt{x+1}\, dx \le 8 \cdot 3$

b) $$\int_0^8 \sqrt{x+1}\, dx = \int_1^9 \sqrt{u}\, du$$

$(u = x+1)$

$$= \frac{2}{3} u^{\frac{3}{2}} \Big|_1^9 = \frac{2}{3}\left[9^{\frac{3}{2}} - 1\right]$$

$$= \frac{2}{3}[27 - 1] = \frac{2}{3} \cdot 26 = \frac{52}{3}$$