~~MAT 22A Final Exam~~ SAMPLE EXAM

Dec 15, 2005

Class #: ____________________ LAST Name (Print): ________________

Student ID #: ________________________ First Name (Print): ___________________

Instructions: ( **Do not open your test until you are told to begin**)

1. Use a pen to PRINT your name in the space above.
   When you open your test before starting your exam, use a pen to PRINT your name on the top right corner of each page.
2. No notes, books or calculators are permitted.
3. Show all your work on these pages. Unsupported answers will receive no credit.
   Try to be neat and accurate. Please clearly mark your answer for each problem.
4. You are expected to do your own work and encourage others to do so as well.
5. If you have any question, please raise your hand.
6. You have untle 3:30 o'clock sharp to finish the exam.

Please read and sign:

I affirm that I neither gave nor received assistance on this examination. All the work that appears on the following pages is solely my own.

Signature: ______________________________

| Problem | Points | Score |
|---|---|---|
| 1 | 10 | |
| 2 | 10 | |
| 3 | 10 | |
| 4 | 10 | |
| 5 | 10 | |
| 6 | 10 | |
| 7 | 10 | |
| 8 | 10 | |
| 9 | 10 | |
| Total | 90 | |

# Problem 1

True-False Questions: Correct answer 2 points, wrong answer $-1$ point, no answer 0 pts.

a) $|u \cdot v| \geq \|u\| \|v\|$ for any two vectors $u$ and $v$ in $\mathbb{R}^n$.

b) $A - Id$ is singular if and only if $-1$ is an eigenvalue of $A$

c) One can find three unit vectors in $\mathbb{R}^5$ that are orthogonal to each other and linearly dependent

d) The space of continuous functions that vanish at $x = 10$ is a vector space

e) $$\det \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 5 & 6 & 7 & 8 & 9 \\ 5 & 6 & 7 & 8 & 9 & 10 \\ 6 & 7 & 8 & 9 & 10 & 11 \end{bmatrix} = 147$$

## Problem 2

(10 points)

a) Calculate

$$\det \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 1 & 1 \\ 2 & 0 & 0 & 2 \\ 0 & 5 & 1 & 0 \end{bmatrix}$$

b) Find the row space of the matrix in a)

c) How many solutions does the linear system written below have

$$\begin{cases} x_1 + 2x_2 + 3x_3 = 1 \\ x_2 + x_3 + x_4 = 2 \\ 2x_1 + 2x_4 = 3 \\ 5x_2 + x_3 = 4 \end{cases}$$

## Problem 3

(10 points)

Please construct (if possible) a $2 \times 2$ matrix with the eigenvalues 4 and 5, and the associated eigenvectors

$$\begin{bmatrix} 3 \\ 4 \end{bmatrix} \text{ and } \begin{bmatrix} 4 \\ 5 \end{bmatrix}.$$

# Problem 4

(10 points)

Let $T$ be a mapping from $\mathbb{R}^3$ into itself given by the formula

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 3x_1 - x_3 \\ 15x_2 - x_3 \\ -10x_2 + 2x_1 \end{bmatrix}$$

a) Is $T$ a linear transformation?

b) Consider the range of $T$ (i.e. the collection of vectors in $\mathbb{R}^3$ of the form $\begin{bmatrix} 3x_1 - x_3 \\ 15x_2 - x_3 \\ -10x_2 + 2x_1 \end{bmatrix}$).

Is it a subspace of $\mathbb{R}^3$?

If the answer is Yes, find its dimension.

(10 points)

# Problem 5

Let $A = \begin{bmatrix} 3 & 1 & -2 \\ -2 & 2 & 2 \\ 0 & 1 & 1 \end{bmatrix}$.

a) Show that the vector $v = (1,1,1)$ is an eigenvector of $A$ and find the eigenvalue corresponding to $v$.

b) Calculate the characteristic polynomial of $A$

c) Find two other eigenvalues of $A$.

(10 points)

## Problem 6

Let $v_1 = \begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix}$, $v_2 = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ -\frac{1}{\sqrt{2}} \end{bmatrix}$, and

$$v_3 = \begin{bmatrix} \frac{1}{\sqrt{6}} \\ -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \end{bmatrix}.$$

a) Show that $\{v_1, v_2, v_3\}$ is a basis in $\mathbb{R}^3$

b) Write vector $w = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$ as a linear combination of $v_1$, $v_2$, and $v_3$

c) Is it true that $\{v_1, w, v_3\}$ is an orthogonal basis in $\mathbb{R}^3$?

(10 points)

# Problem 7

Consider the set of continuous functions on the real line that vanish at all integers.

a) Is it a vector space?

b) If the answer is yes, find its dimension.

If the answer is NO, esplain how you can modify the example in a) to make it into a vector space.

# Problem 8

(10 points)

a) Use Gauss-Jordan reduction process to find the solutions of the following linear system

$$\begin{cases} x_1 + 5x_2 - 7x_3 = 1 \\ 2x_1 - x_2 + x_3 = 1 \\ -x_1 + x_2 - x_3 = -1 \end{cases}$$

b) Find the nullity of the matrix

$$\begin{bmatrix} 1 & 5 & -7 \\ -1 & 1 & -1 \end{bmatrix}.$$

(10 points)

# Problem 9

a) Use the Gram - Schmidt process to orthonormalize the basis

$w_1 = (1, 0, -5)$, $w_2 = (2, 1, 1)$, $w_3 = (1, 0, 1)$

in $\mathbb{R}^3$

b) Find the inverse of

$$A = \begin{bmatrix} \sqrt{2} & \sqrt{3} & 1 \\ \sqrt{2} & 0 & -2 \\ \sqrt{2} & \sqrt{3} & 1 \end{bmatrix}$$