[redacted]

Math 22A(002)

[redacted]

Print your name........................
class # : ....

1. [redacted] Solve the following linear system:

$$\begin{cases} x_1 + 4x_2 - 2x^3 = 7 \\ 3x_1 + 2x_2 + 4x_3 = -9 \end{cases}$$

2. [redacted] Let $A = \begin{bmatrix} 3 & 0 & -1 \\ -1 & 1 & -3 \\ 3 & -1 & 2 \end{bmatrix}$. Use row operations to find A^{-1}.

3. [redacted] **a)** Let $L : \mathbf{R}^3 \longrightarrow \mathbf{R}^2$ be a linear transformation so that $L((1,0,0)) = (2,1)$, $L((0,1,0)) = (-1,2)$ and $L((0,0,1)) = (3,-2)$ Find $L(3,2,-1)$.

[redacted] **b)** Find the standard matrix of the linear transformation T Where $T(x_1, x_2) = (2x_1 - x_2, x_2, -x_1, 3x_1 - 5x_2)$.

4. [redacted] Let $A = \begin{bmatrix} 1 & -2 \\ -2 & 4 \\ -1 & 5 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 3 & 4 \\ 3 & -2 & 1 \end{bmatrix}$ and $C = AB$.

a) Find $col_2(C)$ and express it in two forms as

i) a product of two matrices

ii)a linear combination of appropriate vectors.

b) Find LU decomposition of A.

5. [redacted] a) Carefully state the Cauchy-Schwarz Inequality :

b) Use the Cauchy-Schwarz Inequality to prove Triangular inequality

$$||\mathbf{u}+\mathbf{v}|| \leq ||\mathbf{u}|| + ||\mathbf{v}||$$

c) Find all constants k such that the vectors $\mathbf{u} = (\mathbf{k+1, 1, k^2})$ and $\mathbf{v} = (\mathbf{-6, k, 1})$ be orthogonal vectors.

6. [redacted]

Let $A = \begin{bmatrix} 5 & 1 & 2 \\ 0 & 1 & 0 \\ 2 & -1 & 1 \end{bmatrix}$

a) Use the cofactor expansion about the third row to find $\det(A)$.

b) Find C_{13} and C_{31} the $(1,3)$ and $(3,1)$ of the cofactor matrix C of A. Use part (a) and C_{13} or C_{31}, to find the entry $(1,3)$ of A^{-1}.

7. ([redacted]) a)Carefully list nonsingular Equivalences

1. 2. 5.

4. 5. 6.

b) Prove that if A is invertible, then $\det(A) \neq 0$

8. [redacted] a) Find a matrix A so that $(3I - 2(A^T)^{-1})^T = \begin{bmatrix} 4 & 2 \\ 1 & 1 \end{bmatrix}$

9. [redacted] True-False questions:

TRUE FALSE *If A is an $n \times n$ matrix and $\det(A) = 0$ then $AX = b$ has only trivial solution.*

TRUE FALSE *Any consistent linear system with at least three solutions has infinitely many solutions.*

TRUE FALSE *If $L : R^n \to R^m$ is a linear transformation then $L(\mathbf{0}) = \mathbf{0}$.*

TRUE FALSE *If A and B are 4×4 matrices with $A^2 = A$, and $\det(B) = 2$, then* $\det(2(B^T)^{-1}A^2) = 1$

TRUE FALSE *If $\mathbf{u}$ and $\mathbf{v}$ are solutions of $AX = 0$ then $\mathbf{w} = 3\mathbf{u} + 4\mathbf{v}$ is a solution $AX = b$.*