Problem 1.
Exercise 5.2.13 from page 239.

Problem 2.
Exercise 5.2.14 from page 239.

Problem 3.
Exercise 5.3.1 from page 240.

Problem 4.
Exercise 5.3.2 from page 240.

Problem 5.
Exercise 5.3.3 from page 240.

Problem 6.
Exercise 5.3.4 from page 240.

Problem 7.
Read the subsection b. Polya’ Urn Scheme of the Section 5.4.3 and prove that in the case of general values of $g, r,$ and $c$ the distribution of $X_\infty$ has the density

$$\Gamma((g + r)/c) x^{g/c-1} (1 - x)^{r/c-1}.$$

Hint: Use the asymptotics of the Gamma function $\Gamma(x) = \int_0^{+\infty} e^{-t}t^{x-1}dt$ for large values of $x$

$$\Gamma(x) = \sqrt{2\pi} x^{x-1/2} e^{-x} (1 + o(1))$$

and the relation $\Gamma(x + 1) = x\Gamma(x)$.

Problem 8.
Let $X$ be a non-negative integer valued random variable with the finite $k-$th moment, i.e. $E|X|^k < \infty$. Prove that the generating function $\phi(s) = Es^X$, $s \in [-1, +1]$, has the first $k$ left derivatives at the point $s = 1$ and calculate these derivatives.