## SOLUTIONS

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1. Problem: (You do not need to simplify your answers in Problem 1!)  
(a) Compute the derivative of the following function \( y = \frac{\sqrt[3]{x^3 + 1}}{3x} \).  
\[ y' = \frac{\frac{1}{3} (x^3 + 1)^{-\frac{2}{3}} \cdot 9x^3 - 3(x^3 + 1)^{\frac{1}{3}}}{(3x)^2} \]  
(b) Compute the derivative of the following function \( y = x \sin(\sqrt{x + 1}) \).  
\[ y' = \sin(\sqrt{x + 1}) + x \cos(\sqrt{x + 1}) \cdot \frac{1}{2} (x + 1)^{-\frac{1}{2}} \]  
(c) Find the second derivative of the function \( f(x) = \frac{e^{-x}}{\cos x} \).  
\[ f'(x) = \frac{-e^{-x} \cos x + e^{-x} \sin x}{\cos^2 x} \]  
\[ f''(x) = \frac{e^{-x} \cos x - e^{-x} \sin x}{\cos^4 x} + \frac{-e^{-x} \sin x - e^{-x} \cos x}{\cos^4 x} \]  
\[ = \frac{-e^{-x} 2 \sin x}{\cos^4 x} \]
2. **Problem:** At time $t$, the position $(x, y)$ of a particle moving on a plane is given by $x = t^2 - t, y = t^2 - 2t$. Find the tangent to the curve on which the particle is moving at $t = 3$.

\[
\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t - 2}{2t - 1}
\]

At $t = 3$: slope is $\frac{6 - 2}{6 - 1} = \frac{4}{5}$. Furthermore, at $t = 3$, $x = 6, y = 3$, hence the tangent is

\[
y - 3 = \frac{4}{5}(x - 6), \text{ hence } y = \frac{4}{5}x - \frac{9}{5}.
\]

- **13 points** for computation of slope of tangent
- **7 points** for equation of tangent line
3. Problem:

(a) At which points in the interval \((-2, 2)\) is the function, whose graph is shown below, not differentiable?

The function is not differentiable at \(x = -1, x = 0, x = 1\).

(b) What is the slope of the tangent line to the curve \(f(x) = \cos(x) + 2x\) at the point \((0, 1)\)?

Slope of tangent is equal to derivative of \(f\) at \(x = 0\).

\[
f'(x) = -\sin(x) + 2
\]

\[
f'(0) = -0 + 2 = 2
\]

hence slope of tangent at \((0, 1)\) is 2.
4. Problem:
Assume a bullet fired straight up from the surface of the Earth reaches a height of \( s = 320t - 16t^2 \) feet after \( t \) seconds.

(a) How long will the bullet be aloft?

(b) How high will the bullet go?

(c) What is the speed of the bullet when it hits the ground?

(a) \( s = 0: 320t - 16t^2 \), possible solutions are \( t = 0 \) or \( t = 20 \). Thus the bullet is 20 seconds aloft.

(b) \( s' = 320 - 32t \), hence \( s' = 0 \) at \( t = 10 \)

\[ s(10) = 3200 - 1600 = 1600, \] the bullet will go 1600 feet high.

(c) \( v = s' = 320 - 32t \) At \( t = 20: v = 320 - 640 = -320 \). Speed is \( |v| \), hence the speed is 320 ft/second.
5. **Problem:** Using implicit differentiation, compute $\frac{dy}{dx}$ for

$$e^{2x} = \sin(x + 3y).$$

\[
\frac{d}{dx}(e^{2x}) = \frac{d}{dx}(\sin(x + 3y)).
\]

\[
2e^{2x} = \cos(x + 3y) \cdot (1 + 3\frac{dy}{dx})
\]

\[
\frac{dy}{dx} = \frac{2e^{2x}}{3\cos(x + 3y)} - \frac{1}{3}.
\]