21A SAMPLE MIDTERM EXAM SOLUTIONS

Note: these solutions contain mostly only the answers, but in your exam you must show how you obtain those answers!

1. asymptotes are shown as dashed lines in the graph
   horizontal asymptote: \( y = 1 \)
   vertical asymptotes: \( x = 0, x = -1 \)
   intercepts: \( (1,0) \)

(my sketch does not contain intercepts, but your sketch has to display them).

2. (a) \( 1/3 \)
   (b) \( 3 \)
   (c) \( -\infty \)
   (d) \( -\infty \)

3. (a) yes
   (b) no, because \( \lim_{x \to 0^+} f(x) \neq \lim_{x \to 0^-} f(x) \)

4. Using the fact that \( L = 2/3 \) we get \( D > \frac{5}{9\varepsilon} - \frac{2}{3} \).

5. \[
y = \left| \frac{x - 2}{x^2 - 2} \right| = \left| \frac{x - 2}{(x - \sqrt{2})(x + \sqrt{2})} \right|
\]
Hence the domain of $f$ is the whole real line except $\pm \sqrt{2}$. We can consider $f$ as $f = g \circ h$ where $h$ is the rational function $h(x) = \frac{x-2}{(x-\sqrt{2})(x+\sqrt{2})}$ and $g(y) = |y|$. By the theorem on composition of continuous functions and since rational functions are continuous on their domain, $f$ is continuous on its domain.