21D SAMPLE FINAL EXAM

1. Find $\mathbf{T}, \mathbf{N}$, $\mathbf{B}$, and κ for the space curve $\mathbf{r}(t) = (3\sin t)\mathbf{i} + (3\cos t)\mathbf{j} + 4t\mathbf{k}$.

2. Use a parametrization to express the area of the surface S as a double integral. Then evaluate the integral. S is given by the lower portion cut from the sphere $x^2 + y^2 + z^2 = 2$ by the cone $z = \sqrt{x^2 + y^2}$.

3. Evaluate the following line integral:

$$\int_C xz\, ds,$$

where C is the line segment from $(3, 0, -1)$ to $(2, 2, 1)$.

4. Find the flux for $\mathbf{F}(x, y) = (xy)\mathbf{i} + x^2\mathbf{j}$ across the loop C given by the ellipse $(\frac{x}{16})^2 + (\frac{y}{9})^2 = 1$.

5. Show that the following vector field is conservative. Then find a scalar function $f(x, y, z)$ satisfying $\mathbf{F} = \nabla f$.

$$\mathbf{F}(x, y, z) = (y\cos z - yze^x)\mathbf{i} + (x\cos z - ze^x)\mathbf{j} + (-xy\sin z - ye^x + 1)\mathbf{k}$$

6. Show that

$$\int_A^B z^2 dx + 2y dy + 2xz dz$$

is path independent.

7. Use Green's Theorem to find the circulation of $\mathbf{F}(x, y) = (x^2 + y^2)\mathbf{i} - 2xy\mathbf{j}$ around the triangle with vertices $(0, 0), (1, 0), (0, 2)$.

8. Use the transformation $u = 3x + 2y$, $v = x + 4y$ to evaluate the integral

$$\int_R (3x^2 + 14xy + 8y^2)\, dxdy$$

for the region R in the first quadrant bounded by the lines $y = -(3/2)x + 1$, $y = -(3/2)x + 3$, $y = -(1/4)x$, and $y = -(1/4)x + 1$.

Solutions:

1.

Answer: $T:(3/5\cos t)\mathbf{i}+(-3/5\sin t)\mathbf{j}+4.5\mathbf{k}, N=(-\sin t)\mathbf{i}-(\cos t)\mathbf{j}$, $B=(4/5\cos t)\mathbf{i}-(4/5\sin t)\mathbf{j}-3/5\mathbf{k}$, $\kappa=3/25$

2.

Answer: $(4+2\sqrt{2})\pi$

3.

Answer:

$$\int_C xzds=\int_0^1(3-t)(-1+2t)\frac{ds}{dt}dt=\int(-2t^2+7t-3)(3)dt=...=-1/2$$

4.

Answer: 0

5.

Answer: Component test shows that vector field is indeed conservative.

$$f(z,y,z)=xy\cos z-yze^x+z+C$$

6.

Answer: Proceed by showing that the field is conservative

7.

Answer: $-8/3$

8.

Answer: $64/5$