Homework 3, due to February 17, 2006

All exercise numbers refer to the book ‘Numerical Linear Algebra” of Trefethen and Bau.

Problem 1: Solve exercise 7.1
Problem 2: Solve exercise 7.4
Problem 3: Solve exercise 9.1
Problem 4: Solve exercise 9.2
Problem 3: Solve exercise 10.1
Problem 5: Solve exercise 10.2
Problem 6: Solve exercise 10.3
Problem 7: Solve exercise 11.3

Problem 8: Write a Matlab function \([Q,R] = \text{c1gs}(A)\) that computes a reduced QR factorization \(A = \hat{Q}\hat{R}\) of a real-valued \(m \times n\) matrix \(A\) with \(m \geq n\) using the classical Gram Schmidt algorithm (whence \text{c1gs}). The output variables are a matrix \(\hat{Q} \in \mathbb{R}^{m \times n}\) with orthonormal columns and an upper triangular matrix \(\hat{R} \in \mathbb{R}^{n \times n}\). Test your algorithm by applying it to a random matrix (generated via \text{randn}) and to the Hilbert matrix (generated via \text{hilb}) of size 10 \times 10, 50 \times 50, and 100 \times 100. Check the accuracy by computing \(\|A - \hat{Q}\hat{R}\|_F\) and \(\|\hat{Q}^*\hat{Q} - I\|_F\) (the latter tests orthogonality of \(\hat{Q}\)).