1.2 B Find a unit vector $\boldsymbol{u}$ in the direction of $\boldsymbol{v} = (3, 4)$. Find a unit vector $\boldsymbol{U}$ that is perpendicular to $\boldsymbol{u}$. How many possibilities for $\boldsymbol{U}$?

Solution For a unit vector $\boldsymbol{u}$, divide $\boldsymbol{v}$ by its length $\|\boldsymbol{v}\| = 5$. For a perpendicular vector $\boldsymbol{V}$ we can choose $(-4, 3)$ since the dot product $\boldsymbol{v} \cdot \boldsymbol{V}$ is $(3)(-4) + (4)(3) = 0$. For a *unit* vector perpendicular to $\boldsymbol{u}$, divide $\boldsymbol{V}$ by its length $\|\boldsymbol{V}\|$:

$$\boldsymbol{u} = \frac{\boldsymbol{v}}{\|\boldsymbol{v}\|} = \left(\frac{3}{5}, \frac{4}{5}\right) \qquad \boldsymbol{U} = \frac{\boldsymbol{V}}{\|\boldsymbol{V}\|} = \left(-\frac{4}{5}, \frac{3}{5}\right) \qquad \boldsymbol{u} \cdot \boldsymbol{U} = 0$$

The only other perpendicular unit vector would be $-\boldsymbol{U} = (\frac{4}{5}, -\frac{3}{5})$.

1.2 C Find a vector $\boldsymbol{x} = (c, d)$ that has dot products $\boldsymbol{x} \cdot \boldsymbol{r} = 1$ and $\boldsymbol{x} \cdot \boldsymbol{s} = 0$ with two given vectors $\boldsymbol{r} = (2, -1)$ and $\boldsymbol{s} = (-1, 2)$.

Solution Those two dot products give linear equations for c and d. Then $\boldsymbol{x} = (c, d)$.

$\boldsymbol{x} \cdot \boldsymbol{r} = 1$	is	$2c - d = 1$	**The same equations as**
$\boldsymbol{x} \cdot \boldsymbol{s} = 0$	is	$-c + 2d = 0$	**in Worked Example 1.1 C**

Comment on n equations for $\boldsymbol{x} = (x_1, \ldots, x_n)$ in n-dimensional space

Section 1.1 would start with columns $\boldsymbol{v}_j$. The goal is to produce $x_1\boldsymbol{v}_1 + \cdots + x_n\boldsymbol{v}_n = \boldsymbol{b}$. This section would start from rows $\boldsymbol{r}_i$. Now the goal is to find $\boldsymbol{x}$ with $\boldsymbol{x} \cdot \boldsymbol{r}_i = b_i$.

Soon the $\boldsymbol{v}$'s will be the columns of a matrix A, and the $\boldsymbol{r}$'s will be the rows of A. Then the (one and only) problem will be to solve $A\boldsymbol{x} = \boldsymbol{b}$.

Problem Set 1.2

1 Calculate the dot products $\boldsymbol{u} \cdot \boldsymbol{v}$ and $\boldsymbol{u} \cdot \boldsymbol{w}$ and $\boldsymbol{u} \cdot (\boldsymbol{v} + \boldsymbol{w})$ and $\boldsymbol{w} \cdot \boldsymbol{v}$:

$$\boldsymbol{u} = \begin{bmatrix} -.6 \\ .8 \end{bmatrix} \qquad \boldsymbol{v} = \begin{bmatrix} 4 \\ 3 \end{bmatrix} \qquad \boldsymbol{w} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$

2 Compute the lengths $\|\boldsymbol{u}\|$ and $\|\boldsymbol{v}\|$ and $\|\boldsymbol{w}\|$ of those vectors. Check the Schwarz inequalities $|\boldsymbol{u} \cdot \boldsymbol{v}| \le \|\boldsymbol{u}\|\,\|\boldsymbol{v}\|$ and $|\boldsymbol{v} \cdot \boldsymbol{w}| \le \|\boldsymbol{v}\|\,\|\boldsymbol{w}\|$.

3 Find unit vectors in the directions of $\boldsymbol{v}$ and $\boldsymbol{w}$ in Problem 1, and the cosine of the angle θ. Choose vectors $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ that make $0°$, $90°$, and $180°$ angles with $\boldsymbol{w}$.

4 For any *unit* vectors $\boldsymbol{v}$ and $\boldsymbol{w}$, find the dot products (actual numbers) of

(a) $\boldsymbol{v}$ and $-\boldsymbol{v}$ (b) $\boldsymbol{v} + \boldsymbol{w}$ and $\boldsymbol{v} - \boldsymbol{w}$ (c) $\boldsymbol{v} - 2\boldsymbol{w}$ and $\boldsymbol{v} + 2\boldsymbol{w}$

5 Find unit vectors $\boldsymbol{u}_1$ and $\boldsymbol{u}_2$ in the directions of $\boldsymbol{v} = (1, 3)$ and $\boldsymbol{w} = (2, 1, 2)$. Find unit vectors $\boldsymbol{U}_1$ and $\boldsymbol{U}_2$ that are perpendicular to $\boldsymbol{u}_1$ and $\boldsymbol{u}_2$.