

Problems 20–25 deal with u, v, w in three-dimensional space (see Figure 1.5b).

- 20 Locate $\frac{1}{3}u + \frac{1}{3}v + \frac{1}{3}w$ and $\frac{1}{2}u + \frac{1}{2}w$ in Figure 1.5b. Challenge problem: Under what restrictions on c, d, e , will the combinations $cu + dv + ew$ fill in the dashed triangle? To stay in the triangle, one requirement is $c \geq 0, d \geq 0, e \geq 0$.
- 21 The three sides of the dashed triangle are $v - u$ and $w - v$ and $u - w$. Their sum is _____. Draw the head-to-tail addition around a plane triangle of $(3, 1)$ plus $(-1, 1)$ plus $(-2, -2)$.
- 22 Shade in the pyramid of combinations $cu + dv + ew$ with $c \geq 0, d \geq 0, e \geq 0$ and $c + d + e \leq 1$. Mark the vector $\frac{1}{2}(u + v + w)$ as inside or outside this pyramid.
- 23 If you look at *all* combinations of those u, v , and w , is there any vector that can't be produced from $cu + dv + ew$? Different answer if u, v, w are all in _____.
- 24 Which vectors are combinations of u and v , and *also* combinations of v and w ?
- 25 Draw vectors u, v, w so that their combinations $cu + dv + ew$ fill only a line. Find vectors u, v, w so that their combinations $cu + dv + ew$ fill only a plane.
- 26 What combination $c \begin{bmatrix} 1 \\ 2 \end{bmatrix} + d \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ produces $\begin{bmatrix} 14 \\ 8 \end{bmatrix}$? Express this question as two equations for the coefficients c and d in the linear combination.

Challenge Problems

- 27 How many corners does a cube have in 4 dimensions? How many 3D faces? How many edges? A typical corner is $(0, 0, 1, 0)$. A typical edge goes to $(0, 1, 0, 0)$.
- 28 Find vectors v and w so that $v + w = (4, 5, 6)$ and $v - w = (2, 5, 8)$. This is a question with _____ unknown numbers, and an equal number of equations to find those numbers.
- 29 Find *two different combinations* of the three vectors $u = (1, 3)$ and $v = (2, 7)$ and $w = (1, 5)$ that produce $b = (0, 1)$. Slightly delicate question: If I take any three vectors u, v, w in the plane, will there always be two different combinations that produce $b = (0, 1)$?
- 30 The linear combinations of $v = (a, b)$ and $w = (c, d)$ fill the plane unless _____. Find four vectors u, v, w, z with four components each so that their combinations $cu + dv + ew + fz$ produce all vectors (b_1, b_2, b_3, b_4) in four-dimensional space.
- 31 Write down three equations for c, d, e so that $cu + dv + ew = b$. Can you somehow find c, d, e for this b ?

$$u = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} \quad v = \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix} \quad w = \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$