

Problem Set 1.1

Problems 1–9 are about addition of vectors and linear combinations.

- 1 Describe geometrically (line, plane, or all of \mathbf{R}^3) all linear combinations of

$$(a) \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \text{ and } \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix} \quad (b) \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \text{ and } \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix} \quad (c) \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} \text{ and } \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix} \text{ and } \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$$

- 2 Draw $v = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$ and $w = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$ and $v + w$ and $v - w$ in a single xy plane.

- 3 If $v + w = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$ and $v - w = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$, compute and draw the vectors v and w .

- 4 From $v = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $w = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, find the components of $3v + w$ and $cv + dw$.

- 5 Compute $u + v + w$ and $2u + 2v + w$. How do you know u, v, w lie in a plane?

These lie in a plane because $w = cu + dv$. Find c and d

$$u = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad v = \begin{bmatrix} -3 \\ 1 \\ -2 \end{bmatrix}, \quad w = \begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix}.$$

- 6 Every combination of $v = (1, -2, 1)$ and $w = (0, 1, -1)$ has components that add to _____. Find c and d so that $cv + dw = (3, 3, -6)$. Why is $(3, 3, 6)$ impossible?

- 7 In the xy plane mark all nine of these linear combinations:

$$c \begin{bmatrix} 2 \\ 1 \end{bmatrix} + d \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \text{with } c = 0, 1, 2 \quad \text{and } d = 0, 1, 2.$$

- 8 The parallelogram in Figure 1.1 has diagonal $v + w$. What is its other diagonal? What is the sum of the two diagonals? Draw that vector sum.

- 9 If three corners of a parallelogram are $(1, 1)$, $(4, 2)$, and $(1, 3)$, what are all three of the possible fourth corners? Draw two of them.

Problems 10–14 are about special vectors on cubes and clocks in Figure 1.4.

- 10 Which point of the cube is $i + j$? Which point is the vector sum of $i = (1, 0, 0)$ and $j = (0, 1, 0)$ and $k = (0, 0, 1)$? Describe all points (x, y, z) in the cube.

- 11 Four corners of this unit cube are $(0, 0, 0)$, $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$. What are the other four corners? Find the coordinates of the center point of the cube. The center points of the six faces are _____. The cube has now many edges?

- 12 *Review Question.* In xyz space, where is the plane of all linear combinations of $i = (1, 0, 0)$ and $i + j = (1, 1, 0)$?