Practice problems, week 3, April 23 – April 27.

These problems are optional, and will not be collected or graded. Please feel free to ask questions about these and any other problems in office hours. Note that integrals in polar coordinates WILL NOT be covered on midterm 1.

(Not covered in the book, see movie on my website) Double Integrals.

(Connection between integrals and sums, switching order of integration, integrating over various domains, switching to polar coordinates).

Suggested problems for double integrals:
1. \[\int_0^1 \int_{e^{-y}}^{y} \frac{y}{x+y} \, dx \, dy\]
2. Write the volume of a cylinder of height \( h \) and radius \( R \) as a double integral using cartesian coordinates.
3. Write the volume of a cylinder of height \( h \) and radius \( R \) as a double integral using polar coordinates.
4. Suppose you slice a cylinder of height 1 and radius 0.5 at a 45° degree angle. Using a double integral, find the volume of the bottom piece.

5. Redo problem 4 for a slice at any angle \( 0 \leq \phi \leq 45^\circ \).

6. Solve the following integral \[\int_0^\infty e^{-x^2} \, dx\]

HINTS: Here’s how to get started: First, square the integral and change one of the dummy variables:
\[I = \int_0^\infty e^{-x^2} \, dx\]
\[I^2 = \left( \int_0^\infty e^{-x^2} \, dx \right)^2\]
\[
\begin{aligned}
&= \left( \int_0^\infty e^{-x^2} \, dx \right) \left( \int_0^\infty e^{-y^2} \, dy \right) \\
&= \int_0^\infty \int_0^\infty e^{-x^2-y^2} \, dx \, dy
\end{aligned}
\]

Now, switch to polar coordinates and solve for \( I^2 \). Remember that you really want to find \( I \) and not \( I^2 \), so you’ll have to take a square root.