Homework groups:

You will complete each of seven homework assignments as part of a three- or four-person group. Group members are assigned randomly from your section and will remain the same for the duration of the quarter. Each group turns in one homework, and each participating group member receives the same grade on the assignment. One member of the group is responsible for writing the homework (the writer), and this writer rotates for every assignment.

**Homework groups work best if:** Each member of the homework group finishes (or honestly attempts) the homework independently. At some appointed time, well before the due date, the group meets and everyone compares answers. Any discrepancies are discussed until a consensus is achieved. The writer notes the group consensus and makes sure she or he understands how to do the problem. After the meeting, but before class, the writer neatly and clearly writes the homework according to the **Homework guidelines**.

**Homework groups don’t work if:** One or more of the members skips meetings; each group member does not honestly attempt the homework prior to the meeting; a consensus in not reached for each assigned problem. *If a group member does not adequately participate in the homework, write a note on the homework and alert your TA. That person will not receive credit.*

Homework guidelines for writers:

(Adapted from the website of Professor Andy Ruina). To get full credit, please do these things on each homework.

1. As a group writer, hand homework in to your TA during section the day it is due. Homework is available via Smartsite Thursday night after section, and is due the following week in section (unless stated otherwise). At the discretion of the TA grading the homework, late homework may or may not be accepted for reduced credit.

2. On the first page of your homework, please do the following to facilitate sorting. On the top left corner, please put the course information, your section, TA, homework number and date, e.g.:

   MAT17C
   Section F1
   TA: Ralph Macchio
   HW 1
   Due April 12, 2018.

   On the top right corner, please put your group number, the names of your group members, with the writer at the top and clearly indicated. Also indicate any non-participating group members, e.g.:

   Group 3
   Jaromir Jagr (writer)
   Sarah Jessica Parker
   Michelle Wie
   James Van der Beek (did not participate)

3. Please put a staple at the top left corner. Folded interlocked corners fall apart. Paperclips fall off.

4. **CITE YOUR HELP.** At the top of each problem, clearly acknowledge all help you got from TAs, faculty, students or any other source (with exceptions for lecture, office hours and the text, which need
not be cited). You could write, for example: “Mary Jones pointed out to me that I had forgotten to divide by three in problem 2,” or “Nadia Chow showed me how to do problem 3 from start to finish,” or “I copied this solution word for word from Jane Lewenstein” or “I found a problem just like this one, number 9, at cheatonyourhomework.com, and copied it,” etc. You will not lose credit for getting and citing such help. Don’t violate academic integrity rules: be clear about which parts of your presentation you did not do on your own. Violations of this policy are violations of the UC Davis Code of Academic Conduct.

5. Your work should be laid out neatly enough to be read by someone who does not know how to do the problem. For most jobs, it is not sufficient to know how to do a problem, you must convince others that you know how to do it. Your job on the homework is to practice this. **Box your answers.**

6. Grading and regrading. We have a reasonable grading a regrading policy, see the syllabus.

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**DUE: April 12, 2018. To be handed in during your section.**

The topic of this homework is: functions of two variables (§10.1) and a little bit of chapter (§10.2), that relates to limits of functions of two variables.

These topics are covered in §10.1–2 in Neuhauser.

Problems 1-8 are all or nothing; there is no partial credit available. Make sure you check your answers carefully, since you will receive no credit even for minor errors. Together, these 8 problems are worth 40 points (five points each).

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**(For problems 1 and 2)** The ideal gas equation of state provides an expression for pressure $P$ as a function of volume $V$ and temperature $T$ for a mole of molecules:

$$P(V, T) = \frac{RT}{V}$$

Where $R = 8.314$ Joules per Kelvin (one Joule equals one KiloPascal-Liter, $kPa \cdot L$), $V$ is measured in Liters and $T$ in Kelvin.

1a. Evaluate $P(2, 290)$.
   b. what is the volume $V$ in Liters?
   c. what is the temperature $T$ in Kelvin?

2a. Evaluate $P(290, 2)$.
   b. what is the volume $V$ in Liters?
   c. what is the temperature $T$ in Kelvin?

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**(For problems 3 and 4)** The ideal gas law is a poor approximation of gases at high pressure, since it neglects interactions between molecules. A better approximation is the Van der Waals equation of state

$$P(V, T) = \frac{RT}{V - b} - \frac{a}{V^2}$$
Where $R = 8.314$ Joules per Kelvin (one Joule equals one KiloPascal-Liter, $kPa \cdot L$) and $a$ and $b$ are constants that vary depending on molecular properties of the gas. Suppose that $a = 1.5kPa \cdot L^2$ (KiloPascal-Liter$^2$) and $b = 1L$.

3a. Evaluate $P(2, 290)$.

b. Comparing this result to your answer for 1a, is the ideal gas law a good approximation for these conditions? (YES/NO)

4a. Evaluate $P(290, 2)$.

b. Comparing this result to your answer for 2a, is the ideal gas law a good approximation for these conditions? (YES/NO)

(For problems 5 and 6) Both the ideal gas and the Van der Waals equations of state describe a 3-D surface. To visualize the two surfaces, we would like to draw a series of level lines (or contours). In this case, these are lines of constant pressure, isobars. To do so, we can rearrange the two equations to be

$$T = \frac{PV}{R}$$

for the ideal gas equation of state and

$$T = \left( P + \frac{a}{V^2} \right) \frac{(V - b)}{R}$$

for the Van der Waals equation of state. Recall that $R = 8.314$ Joules per Kelvin (one Joule equals one KiloPascal-Liter, $kPa \cdot L$) and $a = 1.5kPa \cdot L^2$ (KiloPascal-Liter$^2$) and $b = 1L$.

5. Sketch a set of level lines for the ideal gas equation of state, for $P = 1kPa$, $P = 2kPa$, $P = 3kPa$, $P = 4kPa$ and $P = 5kPa$. Let $V$ vary between 1 and 10 Liters. (*Use R/R-studio to graph the function and sketch the resulting plot).

6. Sketch a set of level lines for the Van der Waals equation of state, for $P = 1kPa$, $P = 2kPa$, $P = 3kPa$, $P = 4kPa$ and $P = 5kPa$. Let $V$ vary between 1 and 10 Liters. (*Use R/R-studio to graph the function and sketch the resulting plot).

7. Is the Van der Waals equation of state (see below) continuous? Why or why not?

$$P(V, T) = \frac{RT}{V-b} - \frac{a}{V^2}$$

8. Compute

$$\lim_{{(x, y) \to (0, 0)}} \frac{xy}{x^2 + y^4}$$

along the lines $y = mx$ where $m \neq 0$. What can you conclude?

Problems 9, 10 and 11 do have partial credit. Together, these 3 problems are worth 60 points (20 points each).
9. **Mass on a spring.** A mass bouncing on a spring is a simple model for a variety of biological systems (simple spring-based models have given some deep insight into human and animal running, e.g. R. Blickhan, “The spring-mass model for running and hopping,” Journal of Biomechanics, 1989).

In its simplest form, the mass moves in one dimension, and we can keep track of its position and velocity as a function of time \(x(t)\) and \(v(t)\), respectively. When there is no friction (or other dissipation), energy is conserved. In other words, the quantity

\[
E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2
\]

is a constant, even though \(x(t)\) and \(v(t)\) both change with time.

In the energy equation (Eq. 1) \(k\) is the spring constant and \(m\) is the mass. For the rest of this problem, you may assume \(k = 2\) and \(m = 2\). You do not have to keep track of units.

a. The function \(E(x,v)\) describes a 3D surface. Sketch a set of evenly-spaced level lines, or contours, of \(E(x,v)\) (i.e. draw, for example, \(E = 1\), \(E = 2\), \(E = 3\), \(E = 4\) and \(E = 5\)). (*Use R/R-studio to graph the functions and sketch the resulting plot).

b. Sketch a set of evenly-spaced slices through \(E(x,v)\) along the \(x\)-coordinate (i.e. draw, for example, \(E(x,-2)\), \(E(x,-1)\), \(E(x,0)\), \(E(x,1)\) and \(E(x,2)\)). (*Use R/R-studio to graph the functions and sketch the resulting plot).

c. Sketch a set of evenly-spaced slices through \(E(x,v)\) along the \(v\)-coordinate (i.e. draw, for example, \(E(-2,v)\), \(E(-1,v)\), \(E(0,v)\), \(E(1,v)\) and \(E(2,v)\)). (*Use R/R-studio to graph the functions and sketch the resulting plot).

d. Based on your answers to a-c, draw a 3-D sketch of \(E(x,v)\), showing some of the level lines and slices in parts a-c. Be sure to label your axes and try to draw things approximately to scale.
10. TREASURE HUNT. California Jones, famed explorer, adventurer and professor of archaeology at UC Davis, is searching for the legendary Golden Monkey of El Dorado, a priceless relic buried somewhere in the Sierras. In the late 1890s, a gold prospector named Curly Fries was rumored to have stumbled across the monkey statuette while hiking to his claim. Unfortunately, Curly was bitten by a rattlesnake and died shortly after his discovery. The priceless statuette was thought to be lost forever.

But California Jones has found Curly Fries’ journal! The journal includes a description of Curly’s discovery of the Golden Monkey, as well as a detailed topographical map of the area (below). California, who never took Math 17C, needs your help.

Figure 1: Topographical map from Curly Fries’ journal. Lines of constant altitude (in meters) are shown. Lines are every 200 meters of elevation, with 1000 meter increments in bold.

Curly’s Journal, Part I: “Climbed up from 2,000 meters in a blizzard. Made a steep ascent in a dry creek-bed, with the land rising to my left and right, in order to shelter myself from the wind. Reached the summit at 3,000 meters as the snow cleared. To my left, the descent appeared gradual; to my right the descent appeared a bit steeper. In front of me, there was a valley and the looming bulk of a mountain as tall as the one upon which I stood.”

a. On the map, draw the first part of Curly’s hike and label it “a.”

Curly’s Journal, Part II: “Descended into the valley. Discovered a river at 1,000 meters. Luckily, I found myself at a narrowing of the river and was able to cross. Set up camp.”

b. On the map, draw the second part of Curly’s hike and label it “b.”

Curly’s Journal, Part III: “Broke camp in the morning and starting ascending, following a ridge. At 2600 meters, found a cave. After moving some brush at the opening of the cave, saw a glint in the darkness. It was a fantastic monkey statuette, crusted in jewels – surely the fabled Golden Monkey of El Dorado”

c. On the map, draw the third part of Curly’s hike and label it “c.” Indicate the location of the statuette with an “X.”
Taking the map and your drawing, California Jones heads out to the mountains. He returns a few days later without the crown. Planning to return, he asks you to draw a plot of altitude as a function of distance along each section of Curly's hike.

d. Draw three separate graphs for sections a, b and c. Indicate, as accurately as you can, how altitude changed as Curly hiked along each segment of his journey. Be sure to label your axes.

11. Please turn in a completed version of worksheet 1.