Homework groups:

You will complete each of seven homework assignments as part of a three- or four-person group. Group members are assigned randomly from your section and will remain the same for the duration of the quarter. Each group turns in one homework, and each participating group member receives the same grade on the assignment. One member of the group is responsible for writing the homework (the writer), and this writer rotates for every assignment.

Homework groups work best if: Each member of the homework group finishes (or honestly attempts) the homework independently. At some appointed time, well before the due date, the group meets and everyone compares answers. Any discrepancies are discussed until a consensus is achieved. The writer notes the group consensus and makes sure she or he understands how to do the problem. After the meeting, but before class, the writer neatly and clearly writes the homework according to the Homework guidelines.

Homework groups don’t work if: One or more of the members skips meetings; each group member does not honestly attempt the homework prior to the meeting; a consensus is not reached for each assigned problem. If a group member does not adequately participate in the homework, write a note on the homework and alert your TA. That person will not receive credit.

Homework guidelines for writers:

(Adapted from the website of Professor Andy Ruina). To get full credit, please do these things on each homework.

1. As a group writer, hand homework in to your TA during section the day it is due. Homework is available via Smartsite Thursday night after section, and is due the following week in section (unless stated otherwise). At the discretion of the TA grading the homework, late homework may or may not be accepted for reduced credit.

2. On the first page of your homework, please do the following to facilitate sorting. On the top left corner, please put the course information, your section, TA, homework number and date, e.g.:

   MAT17C
   Section F1
   TA: Ralph Macchio
   HW 3
   Due April 26, 2018.

   On the top right corner, please put your group number, the names of your group members, with the writer at the top and clearly indicated. Also indicate any non-participating group members, e.g.:

   Group 3
   Jaromir Jagr (writer)
   Sarah Jessica Parker
   Michelle Wie
   James Van der Beek (did not participate)

3. Please put a staple at the top left corner. Folded interlocked corners fall apart. Paperclips fall off.

4. CITE YOUR HELP. At the top of each problem, clearly acknowledge all help you got from TAs, faculty, students or any other source (with exceptions for lecture, office hours and the text, which need
not be cited). You could write, for example: “Mary Jones pointed out to me that I had forgotten to divide by three in problem 2,” or “Nadia Chow showed me how to do problem 3 from start to finish,” or “I copied this solution word for word from Jane Lewenstein” or “I found a problem just like this one, number 9, at cheatonyourhomework.com, and copied it,” etc. You will not lose credit for getting and citing such help. Don’t violate academic integrity rules: be clear about which parts of your presentation you did not do on your own. Violations of this policy are violations of the UC Davis Code of Academic Conduct.

5. Your work should be laid out neatly enough to be read by someone who does not know how to do the problem. For most jobs, it is not sufficient to know how to do a problem, you must convince others that you know how to do it. Your job on the homework is to practice this. **Box your answers.**

6. Grading and regrading. We have a reasonable grading a regrading policy, see the syllabus.

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**DUE: April 26, 2018. To be handed in during your section.**

The topic of this homework is: linearizing two functions of two variables (and the Jacobian) (§10.4), the gradient vector and directional derivatives (§10.5.3).

These topics are covered in §10.4 and 10.5.3 in Neuhauser.

Problems 1-8 are all or nothing; there is no partial credit available. Make sure you check your answers carefully, since you will receive no credit even for minor errors. Together, these 8 problems are worth 40 points (five points each).

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1. Here is the treasure map from homework 1.

   a) At the four points on the map (that I’ve labeled a,b,c and d), draw the approximate direction of
the gradient vector (for this part, don’t worry about the length).

b) Which one of the gradient vectors would be the longest?

c) Which one of the four gradient vectors would be the shortest?

For problems 2 and 3. The vectors \( \hat{i} \) and \( \hat{j} \) are unit vectors in 2-D. That is,

\[
\hat{i} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \hat{j} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}
\]

(1)

Thus, any vector can be conveniently written in terms of these unit vectors:

\[
v = \begin{bmatrix} v_x \\ v_y \end{bmatrix} = v_x \hat{i} + v_y \hat{j}
\]

2

a. What’s \( v \cdot \hat{i} \)?

b. What’s \( v \cdot \hat{j} \)?

c. What’s \( \hat{i} \cdot \hat{j} \)?

3. The rotation matrix

\[
R(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}
\]

rotates a vector counterclockwise by an angle \( \theta \) without changing its length. That is, \( R(\theta)\hat{i} \) is the same length as the vector \( \hat{i} \) (i.e. it has unit length), but it is oriented \( \theta \) degrees above the horizontal (recall that \( \hat{i} \) is oriented along the horizontal axis).

a. What’s \( \left( R(\theta)\hat{i} \right) \cdot \left( R(\theta)\hat{j} \right) \)?

b. Sketch \( \left( R(\theta)\hat{i} \right) \) and \( \left( R(\theta)\hat{j} \right) \) when \( \theta = 30^\circ \)

4. Write the linearization of the following function about the point \( x = x^* \) and \( y = y^* \), where \( x^* \) and \( y^* \) are constants.

\[
f(x) = \begin{bmatrix} ax^2y \\ xe^{axy} \end{bmatrix}
\]

where \( a \) is a positive constant and \( x = \begin{bmatrix} x \\ y \end{bmatrix} \).

5. Find the Jacobian of the following function.

\[
f(x) = \begin{bmatrix} a \cos(t) \\ xe^{at} \end{bmatrix}
\]

where \( a \) is a positive constant and \( x = \begin{bmatrix} x \\ t \end{bmatrix} \).
For problems 6, 7 and 8, the function

\[ z = g(x, y) = e^{-x^2 - 2y^2} \]

6. Find \( \nabla g(x, y) \).

7. Find the slope at \( x = 1, y = 2 \) in the direction of the vector \( v = \begin{bmatrix} 2 \\ -1 \end{bmatrix} \)
   (in other words, suppose you were standing on the function \( g(x, y) \) at the point \( x = 1, y = 2 \). How steep would it be in the direction of the vector \( v \)?)

8. a) Find a unit vector that points in the steepest uphill direction at the point \( x = 1, y = 1 \).
   b) Sketch this unit vector along with a contour at \( z = e^{-3} \).

Problems 9, 10 and 11 do have partial credit. Together, these 3 problems are worth 60 points (20 points each).

9. Walking Energetics. People tend to walk with a step frequency \( (f) \) and speed \( (v) \) that minimizes energy cost \( (E(v, f)) \).

   a. Below, I’ve drawn four level lines of energy cost: \( E(v, f) = 1, 2, 3 \) and 4. Sketch the energy cost as a function of walking speed \( v \) for the fixed step frequency, \( f^* \). Your answer will give the energy cost as you move along the gray horizontal line \( (E(v, f^*)) \).

   b. If you’ve done your drawing correctly, your curve should have a single minimum. Suppose you know the function \( E(v, f) \). Explain how you would find this minimum.
c. Below, I’ve drawn four level lines of energy cost: \( E(v, f) = 1, 2, 3 \) and 4. Sketch the energy cost as a function of step frequency \( f \) for a fixed walking speed, \( v^* \). Your answer will give the energy cost as you move along the gray vertical line \( (E(v^*, f)) \).

\[ \begin{array}{c}
\text{Walking Speed (v)} \\
\text{Step frequency (f)}
\end{array} \]

\[ f^* \]

\[ E(v, f) \]

\[ v^* \]

d. If you’ve done your drawing correctly, your curve should have a single minimum. Suppose you know the function \( E(v, f) \). Explain how you would find this minimum.

e. At a fixed walking speed, all possible step frequencies are described by a vertical line (see the gray line in the figure for part c). With reference to the gradient vector, explain why the minimum energy cost occurs when this line is tangent to a contour.

HINT: Recall that the gradient is perpendicular to contours and use your answer to part d.

10. PROPERTIES OF THE GRADIENT VECTOR. On Smartsite, read the handout about the gradient (it can be found under Resources, in a folder entitled “Handouts.” The handout is called “Gradient.pdf”).

Write up solutions to Exercises 1, 2 and 3 on page 3 of the handout.

11. Please turn in a completed version of worksheet 3.