Homework groups: You will complete each of seven homework assignment as part of a three- or four-person group. Group members are assigned randomly and will remain the same for the duration of the quarter. Each group turns in one homework, and each participating group member receives the same grade on the assignment. One member of the group is responsible for writing the homework (the writer), and this writer rotates for every assignment.

Homework groups work best if: Each member of the homework group finishes (or honestly attempts) the homework independently. At some appointed time, well before the due date, the group meets and everyone compares answers. Any discrepancies are discussed until a consensus is achieved. The writer notes the group consensus and makes sure she or he understands how to do the problem. After the meeting, but before class, the writer neatly and clearly writes the homework according to the Homework guidelines (described below).

Homework groups don’t work if: One or more of the members skips meetings; each group member does not honestly attempt the homework prior to the meeting; a consensus in not reached for each assigned problem. If a group member does not adequately participate in the homework, write a note on the homework and alert the TA. That person will not receive credit.

Homework guidelines for writers: (Adapted from the website of Professor Andy Ruina). To get full credit, please do these things on each homework.

1. As a group writer, you must hand homework in by the beginning of class Wednesday, the day it is due. Homework is available via my website Wednesday evenings, and is due the following week in class (unless stated otherwise). Late homework may or may not be accepted for reduced credit.

2. On the first page of your homework, please do the following to facilitate sorting:
   On the top left corner, please put the course information, homework number and date, e.g.:
   MAT207C
   HW 3
   Due April 24, 2019.
   On the top right corner, please put the names of your group members, with the writer at the top and clearly indicated. Non-participating group members should also be indicated, e.g.:
   Jaromir Jagr (writer)
   Sarah Jessica Parker
   Michelle Wie
   James Van der Beek (did not participate)

3. Please put a staple at the top left corner. Folded interlocked corners fall apart. Paperclips fall off.

4. CITE YOUR HELP. At the top of each problem, clearly acknowledge all help you got from TAs, faculty, students or any other source (with exceptions for lecture and the text, which need not be cited). You could write, for example: “Mary Jones pointed out to me that I had forgotten to divide by three in problem 2,” or “Nadia Chow showed me how to do problem 3 from start to finish,” or “I copied this solution word for word from Jane Lewenstein” or “I found a problem just like this one, number 9, at cheatonyourhomework.com, and copied it,” etc. You will not lose credit for getting and citing such help. Don’t violate academic integrity rules: be clear about which parts of your presentation you did not do on your own. Violations of this policy are violations of the UC Davis Code of Academic Conduct.

5. Your work should be laid out neatly enough to be read by someone who does not know how to do the problem. For most jobs, it is not sufficient to know how to do a problem, you must convince others that you know how to do it. Your job on the homework is to practice this. Box your answers.

6. Grading and regrading. We have a reasonable grading and regrading policy (see syllabus).
DUE: Wednesday, April 24, 2019. To be handed to me by the end of class. 
This homework covers matched asymptotic expansions, multiple layers and interior layers.

These topics are covered in §2.2–2.4 in Holmes.

1. (Problem 9 on page 59 from Holmes, 1995) Consider the following equation:

\[ 4\varepsilon \frac{d^2y}{dx^2} + 6\sqrt{x} \frac{dy}{dx} - 3y = -3 \]

for \( 0 < x < 1 \) and \( y(0) = 0, y(1) = 3 \)

a) After finding the first term of the inner and outer expansions, derive a composite expansion for the solution to this problem.

b) Solve the differential equation numerically using Matlab for your choice of \( \varepsilon \) (be sure to list your choice) and compare this numerical solution to your approximations.

EXTRA CREDIT: If you’re feeling energetic, try finding a two term expansion.

2. (Problem 1a on page 65 from Holmes, 1995) Consider the following equation:

\[ \varepsilon \frac{d^2y}{dx^2} + \varepsilon(x + 1)^2 \frac{dy}{dx} - y = x - 1 \]

for \( 0 < x < 1 \) and \( y(0) = 0, y(1) = -1 \)

a) After finding the first term of the inner and outer expansions, derive a composite expansion for the solution to this problem.

b) Solve the differential equation numerically using Matlab for your choice of \( \varepsilon \) (be sure to list your choice) and compare this numerical solution to your approximations.

3. (Problem 1i on page 66 from Holmes, 1995) Consider the following equation:

\[ \varepsilon \frac{d^2y}{dx^2} + y \left( \frac{dy}{dx} \right)^2 - y = 0 \]

for \( 0 < x < 1 \) and \( y(0) = 3, y(1) = 1 \)

a) After finding the first term of the inner and outer expansions, derive a composite expansion for the solution to this problem.

b) Solve the differential equation numerically using Matlab for your choice of \( \varepsilon \) (be sure to list your choice) and compare this numerical solution to your approximations.
4. A LIMIT CYCLE.
The following equations describe the behavior of a Van der Pol oscillator (which you may have learned about in 207A):

\[
\varepsilon \frac{dx}{dt} = \left( x - \frac{1}{3} x^3 - y \right) \tag{1}
\]
\[
\frac{dy}{dt} = x \tag{2}
\]

Suppose that \( \varepsilon = 0.001 \) and \( x(0) = 0.5, y(0) = 0 \).

a) To get an idea how such a system works, use Matlab to draw the null clines. Then, use Matlab to numerically solve the ODEs and plot a trajectory. (You should see a stable limit cycle).

b) Now, plot \( x(t) \). You should see a “fast” phase and a “slow” phase. Which ODE describes the “fast” phase (this is something like an inner approximation, \( x_i(t) \))? Which ODE describes the “slow” phase (this is something like an outer approximation, \( x_o(t) \))? 

c) Find an approximation for the first “fast” and the first “slow” phase (you only need one term in your approximations). (Note, you should end up with transcendental equations).

d) Plot the inner and outer approximations along with \( x(t) \) from your numerical simulations. Note, since your equations are transcendental, it’s not so straightforward to plot \( x_o(t) \) and \( x_i(t) \). However, you should be able to solve your equations for \( t \), and plot \( t(x_o) \) and \( t(x_i) \).

5. NON-DIMENSIONALIZATION PRACTICE.

In class, I introduced the following non-dimensional equation

\[
\frac{d\hat{n}}{d\hat{X}} = -\frac{K}{V} \frac{1}{\varepsilon \sqrt{\pi}} \exp \left( -\left( \frac{X - 1}{\varepsilon} \right)^2 \right) (1 - N) + \frac{1}{V} \exp(-EX)\hat{n}
\]

which comes from the following dimensional equation:

\[
-v \frac{dn}{dx} = ka \sqrt{\frac{k}{2\pi k_B T}} \exp \left( -\frac{k(x - d)^2}{2k_B T} \right) (1 - N) - k_d \exp \left( -\frac{k\delta x}{k_B T} \right) n
\]

Note also that

\[
N = \int_{-\infty}^{\infty} n \cdot dx = \int_{-\infty}^{\infty} \hat{n} \cdot d\hat{X}
\]

What are all the non-dimensional variables in terms of the dimensional variables? (i.e., find expressions for \( \hat{n}, X, K, V, \varepsilon \) and \( E \) in terms of \( n, x, v, k_a, k, k_B, T, d, k_d \) and \( \delta \)).