Homework groups:

You will complete each of seven homework assignment as part of a three- or four-person group. Group members are assigned randomly from your section and will remain the same for the duration of the quarter. Each group turns in one homework, and each participating group member receives the same grade on the assignment. One member of the group is responsible for writing the homework (the writer), and this writer rotates for every assignment.

Homework groups work best if: Each member of the homework group finishes (or honestly attempts) the homework independently. At some appointed time, well before the due date, the group meets and everyone compares answers. Any discrepancies are discussed until a consensus is achieved. The writer notes the group consensus and makes sure she or he understands how to do the problem. After the meeting, but before class, the writer neatly and clearly writes the homework according to the Homework guidelines.

Homework groups don’t work if: One or more of the members skips meetings; each group member does not honestly attempt the homework prior to the meeting; a consensus in not reached for each assigned problem. If a group member does not adequately participate in the homework, write a note on the homework and alert your TA. That person will not receive credit.

Homework guidelines for writers:

(Adapted from the website of Professor Andy Ruina). To get full credit, please do these things on each homework.

1. As a group writer, hand homework in to your TA during section, the day it is due. Homework is available on my website Thursday night after section, and is due the following week in section (unless stated otherwise). At the discretion of the TA grading the homework, late homework may or may not be accepted for reduced credit.

2. On the first page of your homework, please do the following to facilitate sorting. On the top left corner, please put the course information, your section, TA, homework number and date, e.g.:

   MAT17C  
   Section 4  
   TA: Blake Bortles  
   HW 7  
   Due June 7, 2018.

   On the top right corner, please put your group number, the names of your group members, with the writer at the top and clearly indicated. Also indicate any non-participating group members, e.g.:

   Group 12  
   Tycho Brahe (writer)  
   Dame Judi Dench  
   Rosalind Franklin  
   James Van der Beek (did not participate)

3. Please put a staple at the top left corner. Folded interlocked corners fall apart. Paperclips fall off.

4. CITE YOUR HELP. At the top of each problem, clearly acknowledge all help you got from TAs, faculty, students or any other source (with exceptions for lecture and the text, which need not be cited).
You could write, for example: "Mary Jones pointed out to me that I had forgotten to divide by three in problem 2," or "Nadia Chow showed me how to do problem 3 from start to finish," or "I copied this solution word for word from Jane Lewenstein" or "I found a problem just like this one, number 9, at cheatonyourhomework.com, and copied it," etc. You will not lose credit for getting and citing such help. Don’t violate academic integrity rules: be clear about which parts of your presentation you did not do on your own. Violations of this policy are violations of the UC Davis Code of Academic Conduct.

5. Your work should be laid out neatly enough to be read by someone who does not know how to do the problem. For most jobs, it is not sufficient to know how to do a problem, you must convince others that you know how to do it. Your job on the homework is to practice this. **Box your answers.**

6. Grading and regrading. We have a reasonable grading a regrading policy.
DUE: June 7, 2018. To be handed in during your section.
The topics of this homework are: Counting and Probability (§12.1 and 12.2 of Neuhauser). The topic of problem 10 is coupled non-linear ODEs. It is pretty tough, please ask for help if you need it.

Problems 1-8 are all or nothing; there is no partial credit available. Make sure you check your answers carefully, since you will receive no credit even for minor errors. Together, these 8 problems are worth 40 points (five points each).

For prob. 1 and 2 You may recall that tRNA reads a three nucleotide code on mRNA and substitutes an appropriate amino acid into a newly-forming protein. Why is it three nucleotides (and not one or two)? Recall that there are four possible bases, G, C, U and A. Thus, if each base coded for one amino acid, there could be, at most, four amino acids.

1. If a pair of bases coded for one amino acid, how many amino acids could there be? Recalling that there are 20 amino acids, could this work?

2. If three bases coded for one amino acid, how many amino acids could there be? Recalling that there are 20 amino acids, could this work?

3. How many different ways can you arrange the letters: MATHMOVESME?

4. (Based on tic-tac-toe) How many distinct ways are there of placing 5 X’s and 4 O’s in a 3 × 3 grid?

5. How many different five-card hands have three identically numbered cards (three-of-a-kind) and the remaining two have different numbers (e.g. A♦, A♠, A♠, 2♦, 3♠)?

6. Drawing five cards at random from a deck, what is the probability of drawing three-of-a-kind (the hand described in problem 5).

7. The following letters can be unscrambled to spell precisely one word: NIPPEG. Suppose you arrange the letters randomly. What’s the probability that your random arrangement is the word?

8. Suppose you roll a fair six-sided die 10 times. What’s the probability of rolling exactly three 1’s?
Problems 9, 10 and 11 do have partial credit. Together, these 3 problems are worth 60 points (20 points each).

9. **Linkage.** Linkage is an important concept in genetics. Suppose that you’re studying two genes in a fruitfly.

The first gene \((R\) or \(r)\) determines eye color. Being diploid, the fruitfly has two copies of each gene. A fruitfly with \(RR\) (two copies of the functional \(R\) gene) has red eyes; a fruitfly with \(Rr\) or \(rR\) (one copy of the functional \(R\) gene) has brown eyes; and a fruitfly with \(rr\) (no functional copies of the functional \(R\) gene) has white eyes.

The first gene \((W\) or \(w)\) determines wing shape. A fruitfly with \(WW\), \(Ww\) or \(wW\) (at least one copy of the functional \(W\) gene) has normal wings; a fruitfly with \(ww\) (no functional copies of the functional \(W\) gene) has curly wings.

Chromosomes cross-over during meiosis. If two genes are on the same chromosome, but are far apart, then there is likely to be several cross-over points between the genes. On the other hand, if the two genes are close together, then there is unlikely to be even a single cross-over between the genes. This is the idea of linkage, and it can be used to determine the relative position of genes on a chromosome.

Here’s how it works. Suppose the \(R\) and \(W\) genes are on the same chromosome. Then, if we cross an \(RRWW\) fruitfly with an \(rrww\) fruitfly, all offspring are guaranteed to be \(RrWw\).

Suppose we cross two of these \(RrWw\) flies.

a) If the chromosomes are far apart (unlinked), then the male will produce sperm \(RW, Rw, rW\) and \(rw\) with equal likelihood. The female produces eggs \(RW, Rw, rW\) and \(rw\) with equal likelihood. Suppose that each sperm-egg combination produces a viable new fly. Calculate the probability that the offspring has 1) Red eyes and normal wings; 2) Red eyes and curly wings 3) Brown eyes and normal wings; 4) Brown eyes and curly wings 5) White eyes and normal wings; 6) White eyes and curly wings.

b) If the chromosomes are very close together (tightly linked), then the male will produce sperm \(RW\) and \(rw\) with equal likelihood. The female produces eggs \(RW\) and \(rw\) with equal likelihood. Suppose that each sperm-egg combination produces a viable new fly. Calculate the probability that the offspring has 1) Red eyes and normal wings; 2) Red eyes and curly wings 3) Brown eyes and normal wings; 4) Brown eyes and curly wings 5) White eyes and normal wings; 6) White eyes and curly wings.

c) That is, a male might produce mostly \(RW\) and \(rw\) sperm and rarely produce \(Rw\) and \(rW\) sperm. The female would obviously produce similar eggs. Based on your answer to a) and b), describe (qualitatively) the probability that the offspring has 1) Red eyes and normal wings; 2) Red eyes and curly wings 3) Brown eyes and normal wings; 4) Brown eyes and curly wings 5) White eyes and normal wings; 6) White eyes and curly wings.

Mapping works as follows. Suppose you know that genes \(R, W, A\) and \(B\) are all on the same chromosome. You would perform the crosses above, and then consider each pair of genes (e.g. \(R\) and \(W\)). Then, by looking
at the offspring, you’d figure out how linked they are. So, supposing you find R is strongly linked to W and weakly linked to A and unlinked to A and B. You might enter this into a table:

<table>
<thead>
<tr>
<th>Gene</th>
<th>R</th>
<th>W</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>–</td>
<td>strong</td>
<td>weak</td>
<td>0</td>
</tr>
<tr>
<td>W</td>
<td>strong</td>
<td>–</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>A</td>
<td>weak</td>
<td>0</td>
<td>–</td>
<td>strong</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>0</td>
<td>strong</td>
<td>–</td>
</tr>
</tbody>
</table>

Table 1:

Based on this table, you would generate a map that is consistent with your measurements, e.g. WR – AB.

10. Gene Switch. You may recall learning about a gene switch in 17B (if not, ask me about it!). The concentration of protein p was governed by an equation of the form

\[
dp\over dt = -p + 6\frac{p^2}{1+p^2}
\]

a. Use R/R-studio or a graphing calculator to make a plot of \(dp/dt\) as a function of \(p\) and, recalling what you learned in 17B, use graphical stability analysis to sketch a 1D phase portrait (note that I reminded you of this in class). Make sure to
i. indicate all fixed points and their stability.
ii. Indicate flow direction on the horizontal axis.

Now, consider the equation, which models a gene with some constant background expression

\[
dp\over dt = -p + 6\frac{p^2}{1+p^2} + 0.2
\]

b. Use R/R-studio or a graphing calculator to make another plot of \(dp/dt\) as a function of \(p\) and use graphical stability analysis to sketch a 1D phase portrait. Make sure to
i. indicate all fixed points and their stability.
ii. Indicate flow direction on the horizontal axis.
iii. Indicate where the fixed points are relative to the fixed points you found in part a.

Now, consider the coupled equations which model a gene switch that’s regulated by another protein

\[
\begin{align*}
\frac{dp}{dt} &= -p + 6\frac{p^2}{1+p^2} + b \\
\frac{db}{dt} &= -bp + 0.2p
\end{align*}
\]

c. Sketch the full 2D phase plot. Indicate all fixed points, their stability, and their type (i.e. star, saddle, spiral, etc.).

Hints:
1. Your answers to part b. and c. will help you.
2. It will be useful to look for null-clines – that is, lines along which \(\frac{dp}{dt}\) or \(\frac{db}{dt}\) is zero.
3. The equation for \(\frac{db}{dt}\) will be helpful when you look for fixed points.
11. Turn in a completed version of Worksheet 9.