Homework groups: You will complete each of seven homework assignments as part of a three- or four-person group. Group members are assigned randomly and will remain the same for the duration of the quarter. Each group turns in one homework, and each participating group member receives the same grade on the assignment. One member of the group is responsible for writing the homework (the writer), and this writer rotates for every assignment.

Homework groups work best if: Each member of the homework group finishes (or honestly attempts) the homework independently. At some appointed time, well before the due date, the group meets and everyone compares answers. Any discrepancies are discussed until a consensus is achieved. The writer notes the group consensus and makes sure she or he understands how to do the problem. After the meeting, but before class, the writer neatly and clearly writes the homework according to the Homework guidelines (described below).

Homework groups don’t work if: One or more of the members skips meetings; each group member does not honestly attempt the homework prior to the meeting; a consensus is not reached for each assigned problem. If a group member does not adequately participate in the homework, write a note on the homework and alert the TA. That person will not receive credit.

Homework guidelines for writers: (Adapted from the website of Professor Andy Ruina). To get full credit, please do these things on each homework.

1. As a group writer, you must hand homework in by the end of class Monday, the day it is due. Homework is available on my website Monday evenings, and is due the following week in class (unless stated otherwise). Late homework may or may not be accepted for reduced credit.

2. On the first page of your homework, please do the following to facilitate sorting:
   On the top left corner, please put the course information, homework number and date, e.g.:
   MAT207A
   HW 5
   Due November 13, 2017.

   On the top right corner, please put the names of your group members, with the writer at the top and clearly indicated. Non-participating group members should also be indicated, e.g.:
   Martin Riggs (writer)
   Whoopi Goldberg
   Jane Lewenstein
   Roger Murtaugh (did not participate)

3. Please put a staple at the top left corner. Folded interlocked corners fall apart. Paperclips fall off.

4. CITE YOUR HELP. At the top of each problem, clearly acknowledge all help you got from TAs, faculty, students or any other source (with exceptions for lecture and the text, which need not be cited). You could write, for example: “Mary Jones pointed out to me that I had forgotten to divide by three in problem 2,” or “Nadia Chow showed me how to do problem 3 from start to finish,” or “I copied this solution word for word from Jane Lewenstein” or “I found a problem just like this one, number 9, at cheatonyourhomework.com, and copied it,” etc. You will not lose credit for getting and citing such help. Don’t violate academic integrity rules: be clear about which parts of your presentation you did not do on your own. Violations of this policy are violations of the UC Davis Code of Academic Conduct.

5. Your work should be laid out neatly enough to be read by someone who does not know how to do the problem. For most jobs, it is not sufficient to know how to do a problem, you must convince others that you know how to do it. Your job on the homework is to practice this. Box your answers.

6. Grading and regrading. We have a reasonable grading and regrading policy (see syllabus).
DUE: Monday, November 13, 2017. To be handed to me by the end of class.
The topics of this homework are 1. Conservative systems; 2. Limit cycles; and 3. Lyapunov functions, ruling
out limit cycles.

These topics are covered in §6 and §7 in Strogatz.

1. Problem 6.5.1.
   a. Consider the system
      \[ \ddot{X} = X^3 - X \]
      Find all fixed points and classify them. Find a conserved quantity. Sketch the phase portrait.
   b. Check your answers by generating a phase portrait with Matlab. (Simulate several initial conditions, and
      plot \( \dot{X} \) and \( X \))

2. Problem 6.5.19.

   The Lotka-Volterra predator-prey model, describes the evolution of a population of rabbits and foxes. In
   non-dimensional form, it is
   \[
   \begin{align*}
   \dot{R} &= R - RF \\
   \dot{F} &= -\alpha F + \beta RF
   \end{align*}
   \]
   \( R(T) \) is the number of rabbits, \( F(T) \) the number of foxes (naturally, these cannot be negative). \( \alpha, \beta > 0 \) are
   parameters.

   Show that the model predicts cycles in the populations of both species for almost all initial conditions (you
   do not need to be overly rigorous, but try to construct a convincing argument – Hint: there is a conserved
   quantity).

3. Consider the following differential equation
   \[ \ddot{x} = f(x) - (\dot{x})^n \]
   where \( n \) is a positive, odd integer.

   a. Show that closed orbits do not exist.
   b. Do closed orbits exist for the following system (again, assuming \( n \) is a positive, odd integer)?
      \[ \ddot{x} = f(x) + (\dot{x})^n \]
      Why or why not?

4. Consider the following differential equation
   \[ \ddot{x} = -(x^2 + (\dot{x})^2 - 1) \dot{x} - x \]
a. Do closed orbits exist? Why or why not? (Here, you need not be overly rigorous. Simply make an argument one way or the other).

b. Check your answer using Matlab.