# SHOCK WAVE COSMOLOGY INSIDE A BLACK HOLE: A COMPUTER VISUALIZATION

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**Abstract**

We discuss a computer visualization of the shock wave cosmological model introduced by Smoller and Temple in [12].

## 1 Introduction

Our starting point is the following question: Could there be a wave at the leading edge of the expansion of the galaxies set in motion by the Big Bang? Such a wave would mark the boundary between the region of expanding galaxies and a different spacetime beyond the wave. If the wave exists, then the total mass of the galaxies inside the wave would be finite, and so we ask, could the Big Bang have been an explosion of finite total mass with a wave at the leading edge of the expansion, similar to a classical explosion? In the standard model of cosmology based on the Friedmann universe, the Big Bang is an explosion of infinite mass and infinite extent, (the critical Friedmann universe is infinite at each time after the Big Bang)—so there is nothing beyond the galaxies in the standard model. Now not knowing anything about the spacetime beyond the wave, one might think that any spacetime could be on the other side of it. But if the explosion is large enough, so that the wave is far enough out to be believable, then, since the total mass $M$ inside radius $r$ increases like $r^3$, far enough out we must have $2M/r > 1$, the condition for being inside a black hole in the Schwarzschild spacetime. In this sense, if the wave lies beyond a sufficiently large radial distance, then the whole spacetime lies inside a black hole—and so there are constraints, and not everything is possible. Based on this, we pose the following mathematical question: Is the possibility that the Big Bang was an explosion of finite total mass, and the existence of a consequent wave at the leading edge of the expanding galaxies far enough out to be believable, consistent with the the principles of general relativity? In [12, 13] the authors confirm the consistency of this possibility in an exact solution. The purpose of this talk is to describe the solution in [12], and to present a computer visualization of the cosmological model implied by this solution. (For a longer version with details, see [12, 13, 14]).

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## 2 Cosmology With a Shock Wave Inside a Black Hole

In the standard model of cosmology, the expanding universe of galaxies is described by a Friedmann-Robertson-Walker (FRW) metric, which in spherical coordinates has a line element given by [1, 16, 17],

$$ds^2 = -dt^2 + R^2(t) \left\{ \frac{dr^2}{1-kr^2} + r^2[d\theta^2 + sin^2\theta \; d\phi^2] \right\}. \tag{1}$$

In this model, which accounts for things on the largest length scale, the universe is approximated by a space of uniform density and pressure at each fixed time, and the expansion rate is determined by the cosmological scale factor $R(t)$ that evolves according to the Einstein equations. Astronomical observations show that the galaxies are uniform on a scale of about one billion lightyears, and to within experimental error, the expansion is critical—that is, $k = 0$ in (1)—and so, according to (1), on the largest scale, the universe is infinite flat Euclidian space $R^3$ at each fixed time, with arclenth radial distance give by $\bar{r} = Rr$. Matching the Hubble constant $H = \dot{R}/R$ to its observed values, and invoking the Einstein equations, the FRW model implies that the entire infinite universe $R^3$ emerged all at once from a singularity, (R=0), some 14 billion years ago, and this event is referred to as the Big Bang.

Now in the standard Schwarzschild metric outside a central point mass $M$,

$$ds^2 = -Bd\bar{t}^2 + A^{-1}d\bar{r}^2 + \bar{r}^2 d\Omega^2, \tag{2}$$

$$B = A = 1 - \tfrac{2M}{\bar{r}}, \tag{3}$$

the Schwarzschild radius $\bar{r} = 2M$ plays a critical role, this being the transition from the region outside the black hole ($2M/\bar{r} < 1$) to the region inside the black hole ($2M/\bar{r} > 1$). Thus if there is a wave at the leading edge of the expansion of the galaxies, (which to start we can model as a discrete shock wave), the following natural question presents itself: What is the critical (smallest) radius $\bar{r}_{crit}$ at each fixed time $t > 0$ in a $k = 0$ FRW metric such that the total mass inside a shock wave positioned beyond that radius puts the universe inside a black hole in the sense that $\frac{2M}{\bar{r}} > 1$?[4] The answer is that when $k = 0$, $\bar{r}_{crit}$ is exactly equal to the Hubble length $\bar{r}_{crit} = \frac{c}{H}$, where $c$ denotes the speed of light. To see this, use that the total mass inside radius $\bar{r}$ at fixed time $t > 0$ in the FRW spacetime is equal to the integral at fixed $t$ of the FRW energy density $\rho(t)$,

$$M(\bar{r}, t) = 4\pi \int_0^{\bar{r}} \rho(t)\xi^2 d\xi = \frac{4\pi}{3}\rho(t)\bar{r}^3, \tag{4}$$

and then relate the energy density $\rho(t)$ to the Hubble length through the FRW equation

$$H^2 = \frac{8\pi}{3}\rho, \tag{5}$$

[4]For our purposes here we refer to the region where $2M/\bar{r} > 1$ as the region inside the black hole, independent of the time orientation of the solution. Of course, in an FRW spacetime, this region is relative to the choice of center for the coordinates, but in a finite mass solution the region has geometric significance independent of the center.

which follows from the Einstein equations, [12]. Substituting (5) into (4) we see that the condition $2M/\bar{r} = 1$ is equivalent to $\bar{r}_{crit} = 1/H$, which is $\frac{c}{H}$ when units of time are included. Alternatively, transforming (1) over to standard Schwarzschild coordinates, (coordinates in which the metric takes the general form (2) with $M$ and $B$ as general functions of $(\bar{t}, \bar{r})$), gives

$$ds^2 = -\frac{1}{\psi^2}\left\{\frac{1}{1 - H^2\bar{r}^2}\right\} d\bar{t}^2 + \left\{\frac{1}{1 - H^2\bar{r}^2}\right\} d\bar{r}^2 + \bar{r}^2 d\Omega^2, \tag{6}$$

where $\psi$ is an integrating factor that determines the time coordinate $\bar{t}$, c.f. [12]. Thus the coefficient of $d\bar{r}^2$ is $\frac{1}{1-H^2\bar{r}^2} = \frac{1}{1-2M/\bar{r}}$, and we see that like the Schwarzschild metric, $\bar{r}_{crit} = c/H$ plays the role of a coordinate singularity for the FRW metric in standard Schwarzschild coordinates. In fact, the coordinates $(t, r)$ in (1) regularize the singularity in (6) just as the Eddington-Finkelstein or Kruskall coordinates regularize the Schwarzschild singularity $\bar{r} = 2M$ in (2), c.f. [12]. Of course, the Hubble constant is only approximately constant over small enough time scales, and, in general, $H = H(t)$ is a decreasing funtion, and so the Hubble length $c/H(t)$ is an increasing function of FRW time $t$. The Hubble length $c/H_0$ at time $t_0$ is estimated to be on the order of $10^{10}$ light years.

The Hubble length $c/H$ is a measure of the furthest a viewer can see in the visible universe. That is, $1/H(t)$ is a good estimate of the age of the universe at time $t > 0$ in the standard model, so the Hubble length $c/H$ is the distance that a lightlike signal will travel starting at the Big Bang and ending at time $t$. (Of course, you must choose a coordinate system with respect to which travel distance can be measured, and it turns out that measuring at fixed time in FRW coordinates, the visible universe extends out beyond one Hubble length.) Thus we have the following picture: the radius to which you would have to squeeze a mass to form a black hole is the Schwarzschild radius for that mass. The Schwarzschild radius $\bar{r} = 2M$ for the mass of the earth is on the order of a centimeter, for the mass of the sun on the order of a kilometer, for the mass of a typical galaxy that has some $10^{12}$ solar masses on the order of $10^{12}$ kilometers; and if you ask how far you would have to squeeze all of the mass of the visible universe to form a black hole, the answer is not at all—the Schwarzschild radius of all of the galaxies and matter in the visible universe out to one Hubble length in a $k = 0$ FRW spacetime, is exactly one Hubble length.

The Hubble length as a critical length scale explains the limitations in the authors' previous examples [7, 11] of shock matching outside the black hole, the case when $\frac{2M}{\bar{r}} < 1$, (c.f. Section 6, [11]). In these examples, we obtained exact shock wave solutions of the Einstein equations by matching a $k = 0$ FRW metric to a stationary metric of the form (2) *outside the black hole*, by which we mean that $A$ and $B$ depend only on $\bar{r}$, and $\bar{r} < 2M$, $(A > 0)$, throughout the solution. Stationary, spherically symmetric, matter filled spacetimes based on metrics of form (2) with $A > 0$, describing static fluid spheres in general relativity, were first studied by Tolmann, Oppenheimer and Volkoff, and the Chandresekhar and Bukdahl stability limits in stars are based on an analysis of the Einstein equations for such metrics, [10]. We call a stationary solution of the Einstein equations of form (2) when $A > 0$, a Tolmann-Oppenhemier-Volkoff (TOV) metric *outside the black hole.*

The limitation in our previous work [11] was that we could not find a shock wave matching of a $k = 0$ FRW metric to a TOV metric *outside the black hole* for any shock wave with radial position $\bar{r} > c/H$. We now understand this as follows: we cannot match a critically expanding FRW metric to a classical TOV metric beyond one Hubble length without continuing the TOV solution into the black hole region $\frac{2M}{\bar{r}} > 1$, and we showed in [9] that the standard TOV metric cannot be continued smoothly across the boundary of the black hole $2M/\bar{r} = 1$ except in the limiting case of the Schwarzschild metric, the case when there is no matter present. The purpose of [12] is to show that the total mass of the FRW metric can be cut off by a shock wave positioned out beyond one Hubble length, and that in the resulting solution, the Big Bang reduces to an explosion of finite total mass. The idea is to construct an exact solution of the Einstein equations by matching the $k = 0$ FRW metric to what we call the TOV metric *inside the black hole*, a metric of the form (2) restricted to the case $2M/\bar{r} > 1$. In this case, the character of the metric and the character of the Einstein equations that describe it are different. In particular, when $2M/\bar{r} > 1$, the coordinate $\bar{r}$ is timelike and $\bar{t}$ is spacelike, so rather than describing a stationary metric, the equations describe a metric evloving in time, such that everything is a function of the timelike variable $\bar{r}$. The crucial point is that this TOV metric inside the black hole is not just any old metric, it is exactly what we need, because the total mass $M(\bar{r})$ of the TOV metric beyond the shock interface is constant at each fixed time $\bar{r}$, and since the mass function $M$ is continuous across a shock wave, (c.f. [2, 6, 15]), the TOV metric *inside the black hole* provides the simplest metric that cuts off the total mass to a finite value at each fixed time.

For our solution in [12, 13], we construct a simple class of exact, entropy satisfying shock wave solutions of the Einstein equations for a perfect fluid by matching a $k = 0$ FRW metric to a TOV metric *inside the black hole*, [3, 5]. For simplicity, we assume an FRW equation of state of the form $p = \sigma\rho$, $\sigma = constant$, $0 < \sigma < 1$, [5] and the TOV density and pressure $\bar{\rho}$ and $\bar{p}$ are determined by the equations, subject to the physical condition $0 < \bar{p} < \bar{\rho}$, and the entropy condition for an outward explosion, $\bar{p} < p$, $\bar{\rho} < \rho$. (If we were to impose the TOV equation of state, then other waves like rarefaction waves would be present—a case pretty much impossible to describe globally in closed form). One can view [12] as a natural extension of the Oppeheimer-Snyder (OS) model to the case of non-zero pressure, *inside the black hole*, c.f. [4]. These solutions put forth a new cosmological model in which the expanding Friedmann-Robertson-Walker (FRW) universe emerges from the Big Bang with a shock wave at the leading edge of the expansion, analogous to a classical shock wave explosion, except that, like the standard model of cosmology, the entire spacetime is created at the instant of the Big Bang. Unlike the standard model, however, the Big Bang is an explosion of finite total mass, but it is large enough to account for the enormous scale on which the galaxies and the background radiation appear uniform. In these models, the shock wave must lie beyond one Hubble length from the FRW center, this threshhold being the boundary across which the bounded mass lies inside its own Schwarzschild radius, $2M/r > 1$, and in this sense the shock wave solution evolves inside a black hole. The entropy condition, which breaks the time symmetry by selecting the explosion over the implosion, also implies that the shock wave must weaken until it eventually settles down to a zero pressure OS interface, bounding a *finite* total mass, that emerges from the white hole event horizon of an ambient Schwarzschild spacetime. For each $\sigma$, the total mass of the end state of the explosion is determined by a free parameter in the model, and by choice of this parameter, the mass can be arbitrarily large, and the shock wave arbitrarily far out. One of the interesting surprises is that, unlike shock matching outside black hole, the equation of state $p = \frac{1}{3}\rho$, the equation of state at the earliest stage of Big Bang physics, is *distinguished* at the instant of the Big Bang—for this equation of state alone, the shock wave emerges from the Big Bang at a finite nonzero speed, the speed of light, decelerating to a subluminous wave from that time onward. These shock wave solutions indicate a new cosmological model in which the Big Bang arises from a localized white hole explosion occurring inside a matter filled universe that eventually evolves outward through the white hole event horizon of an asymptotically flat Schwarzschild spacetime.

[5] This catches the equation of state $p = \frac{c^2}{3}\rho$ correct at the earliest stage of Big Bang physics.

More precisely, letting $S = 1/n^2$ where $n$ equals the number of Hubble lengths from the FRW center $\bar{r} = 0$ to the shock wave at time $t > 0$, and letting $u = \frac{\bar{p}}{\rho}$, we show that the TOV metrics that match the given FRW metric Lipschitz continuously across a shock wave $n$ Hubble lengths out, such that there are no delta function sources at the shock, and such that conservation of energy and momentum hold at the shock, are determined by solutions of the non-autonomous ODE for $u$ given by

$$\frac{du}{dS} = \left\{ \frac{(1+u)}{2(1+3u)S} \right\} \left\{ \frac{(3u-1)(\sigma - u) + 6u(1+u)S}{(\sigma - u) + (1+u)S} \right\}. \tag{7}$$

When $\sigma$ is constant, a solution $u(S)$ of (7) determines all other quantities in an FRW-TOV shock wave solution inside the black hole. Here $0 < S < 1$, where $S = 0$ represents the Big Bang, and $S = 1$ marks the time when the shock wave is exactly one Hubble length from the FRW center, the instant when the solution emerges from the white hole. We show that the entropy condition $\bar{p} < p$, $\bar{\rho} < \rho$, and the TOV equation of state bound $0 < \bar{p} < p$ are equivalent to the single condition

$$S < \left( \frac{1-u}{1+u} \right) \left( \frac{\sigma - u}{\sigma + u} \right). \tag{8}$$

Using this we prove the following theorem:

**Theorem 1** *For every $\sigma$, $0 < \sigma < 1$, there exists a unique solution $u_\sigma(S)$ of (7), such that (8) holds on the solution for all $S$, $0 < S < 1$, and on this solution, $0 < u_\sigma(S) < \bar{u}$, $\lim_{S \to 0} u_\sigma(S) = \bar{u}$, where $\bar{u} = Min\{1/3, \sigma\}$, and*

$$\lim_{S \to 1} \bar{p} = 0 = \lim_{S \to 1} \bar{\rho}. \tag{9}$$

Concerning the the shock speed, we have:

**Theorem 2** *Let $0 < \sigma < 1$. Then the shock wave is everywhere subluminal, that is, the shock speed $s_\sigma(S) \equiv s(u_\sigma(S)) < 1$ for all $0 < S \leq 1$, if and only if $\sigma \leq 1/3$. Moreover, concerning the shock speed at the Big Bang $S = 0$, we have*

$$\lim_{S \to 0} s_\sigma(S) = 0, \quad \sigma < 1/3, \tag{10}$$

$$\lim_{S \to 0} s_\sigma(S) = \infty, \quad \sigma > 1/3, \tag{11}$$

$$\lim_{S \to 0} s_\sigma(S) = 1, \quad \sigma = 1/3. \tag{12}$$

## 3 A Computer Visualization

In our cosmological interpretation of the FRW metric, we (loosely) identify the motion of the galaxies with the motion of the FRW fluid, a perfect fluid with nonzero pressure, co-moving with the FRW metric. Assuming this, the shock wave moves *outward* through the galaxies, ($\dot{r} > 0$), and the Hubble length increases with time, but the number of Hubble lengths from the FRW center to the shock wave, as well as the total mass behind the shock wave, both *decrease* in time, tending to infinity in backwards time at the instant of the Big Bang. This is no contradiction because the FRW pressure $p$ is assumed nonzero, c.f. [13]. Since the Hubble length increases with time, more and more galaxies pass inside of the threshold distance of one Hubble length and come into view at the FRW center as time evolves. After the Big Bang, the shock wave in our exact solution continues to weaken as it expands outward, satisfying the entropy condition for shocks all the way out until the Hubble length eventually catches up to the shock wave. At this instant the shock wave lies at the critical distance of exactly one Hubble length from the FRW center. From this time onward, the shock wave can be approximated by a zero pressure, $k = 0$ Oppenheimer-Snyder (OS) interface that emerges from the white hole event horizon of an ambient Schwarzschild metric of finite mass. The entropy condition implies that the TOV density and pressure tend to zero as the shock interface approaches the critical distance of one Hubble length. Thereafter the interface continues on out to infinity along a geodesic of the Schwarzschild metric outside the the black hole. Thus the OS solution gives the large time asymptotics of this new class of shock wave solutions that evolve inside a black hole.

A computer visualization of this evolution was developed by Zeke Vogler as a research project in our Graduate Group in Applied Mathematics at UC-Davis during the summer of 2004. A full color version is soon to be found on our website at *http://www.math.ucdavis.edu/∼temple*, but for purposes of exposition we include several snapshots here. Figure 1.a represents the initial Big Bang. Note here that, as in the standard model, the whole FRW spacetime inside the shock wave, as well as the TOV solution beyond the shock wave, are created at the instant of the Big Bang $t = 0$. The shock wave emerges from the FRW center $\bar{r} = 0$, and the total mass of the explosion is finite at every $t > 0$. In Figure 1.b, we see the solution a short time after the Big Bang. The shock wave is represented by the thick grey, (which is red in the color version), circular region that describes the transition between the inner white region of the FRW metric to the outer black TOV spacetime that lies beyond the outermost circle. The region inside this shock layer represents our FRW expanding universe, and the region beyond this layer represents the TOV spacetime inside the black hole. The inner grey region in Figure 1.b represents the region of the FRW spacetime at the center of the explosion that has recieved no information about the shock wave. The grey region is special to shock wave explosions inside the black hole and is not present in FRW-TOV shock wave models outside the black hole, [7, 11]. At a given time, the inner grey region can be arbitrarily large, and so can persist for an arbitrarily long time, depending on a free parameter in the model that can be interpreted as the total mass of the explosion. Loosely speaking, we could say that for models inside the black hole, information about the shock wave propagates inward from the wave, while outside the black hole information propagates outward from the center of the explosion. The outer boundary of this circular grey region represents an incoming lightlike signal emanating from the shock wave at $t = 0+$, an instant after the Big Bang. It propagates inward until it reaches the center, marking the time when the grey region disappears, and the shock wave is visible to all observers inside. In a sense, the shock wave is able to get far out early on when the spacetime is highly compressed, and after that, the propagation of information communicating its position is restricted by the speed of light bound. The outgoing thin white circle, (which is yellow in the color vesion), represents the points exactly one Hubble length $c/H(t)$ from the FRW center. This curve emanates from the center of the explosion at $t = 0+$, an instant after the Big Bang, and evolves outward through the grey region, and beyond, from that time onward as the Hubble length increases. A calculation shows that the grey region in the center will degenerate to zero, (marking the first time when the shock wave is visible at the FRW center), *before* the Hubble length catches up to the shock wave. Figure 1.c depicts a time after the grey region has disappeared, the white region iniside the shock layer has evolved into galaxies, and the outward propagating white circle representing the Hubble length has not yet caught up to the shock—and so at this stage the whole solution remains inside the black hole. Eventually, the white circle representing the Hubble length will catch up to the shock wave, and at this instant, the wave is exactly one Hubble length from the center of the explosion. This is the critical time $t_{crit}$ at which the solution emerges from the black hole and satisfies $2M/\bar{r} < 1$ from this time onward. Assuming that the pressure is zero after time $t_{crit}$, the solution emerges from the white hole event horizon of a Schwarzschild metric, and evolves as a zero pressure Oppenhiemer-Snyder solution outside the black hole, from time $t_{crit}$ onward. At the end, the solution looks like a finite ball of mass expanding into empty space, outside the black hole, something like a giant supernova. This final stage is represented in Figure 1.d. At this stage, the grey region beyond the shock is the Schwarzschild metric outside the black hole. A calculation shows that the time $t_{crit}$ is no more than about four times the time it takes for the shock wave to become visible at the FRW center, c.f. [12].

One might ask how an observer near the FRW center would first detect evidence of such a cosmic shock wave. Since the shock wave emerges from the Big Bang beyond the Hubble length, the model would imply a uniform expansion throughout a region that is initially well beyond the backward light cone of an observer positioned near the FRW center. If the shock wave were initially far enough out, then the uncoupling of matter from radiation at about $300,000$ years after the Big Bang would produce an extended region with a uniform background radiation field. This region of uniformity would persist until roughly the time when the Hubble length catches up to the shock wave, a time determined by the initial conditions. The influence of the solution beyond the shock wave would propagate into this radiation field at the speed of light, first appearing to an observer that is off center on the FRW side of the shock as a disturbance in the background radiation field at a point in the sky in the direction nearest the shock wave, and this disturbance would grow from that time onward. Since, in our model, the density and pressure are smaller beyond the shock wave, we would expect this disturbance to show itself as a second temperature, lower than the microwave background radiation temperature, in the direction of the shock wave.

# 4 Concluding Remarks

These shock wave solutions of the Einstein equations *inside the black hole* confirm the mathematical consistency of an FRW universe of finite extent and non-zero pressure expanding outward from behind an entropy satisfying shock wave emerging from the origin at subluminal speed beyond one Hubble length at the instant of the Big Bang, a prerequisite for early Big Bang physics. Since the shock wave emerges from the Big Bang beyond one Hubble length, it would account for the uniform thermalization of radiation in an arbitrarily large central region that, for some time, would appear to an observer to be no different from the FRW metric by itself. Surprisingly, unlike shock matching

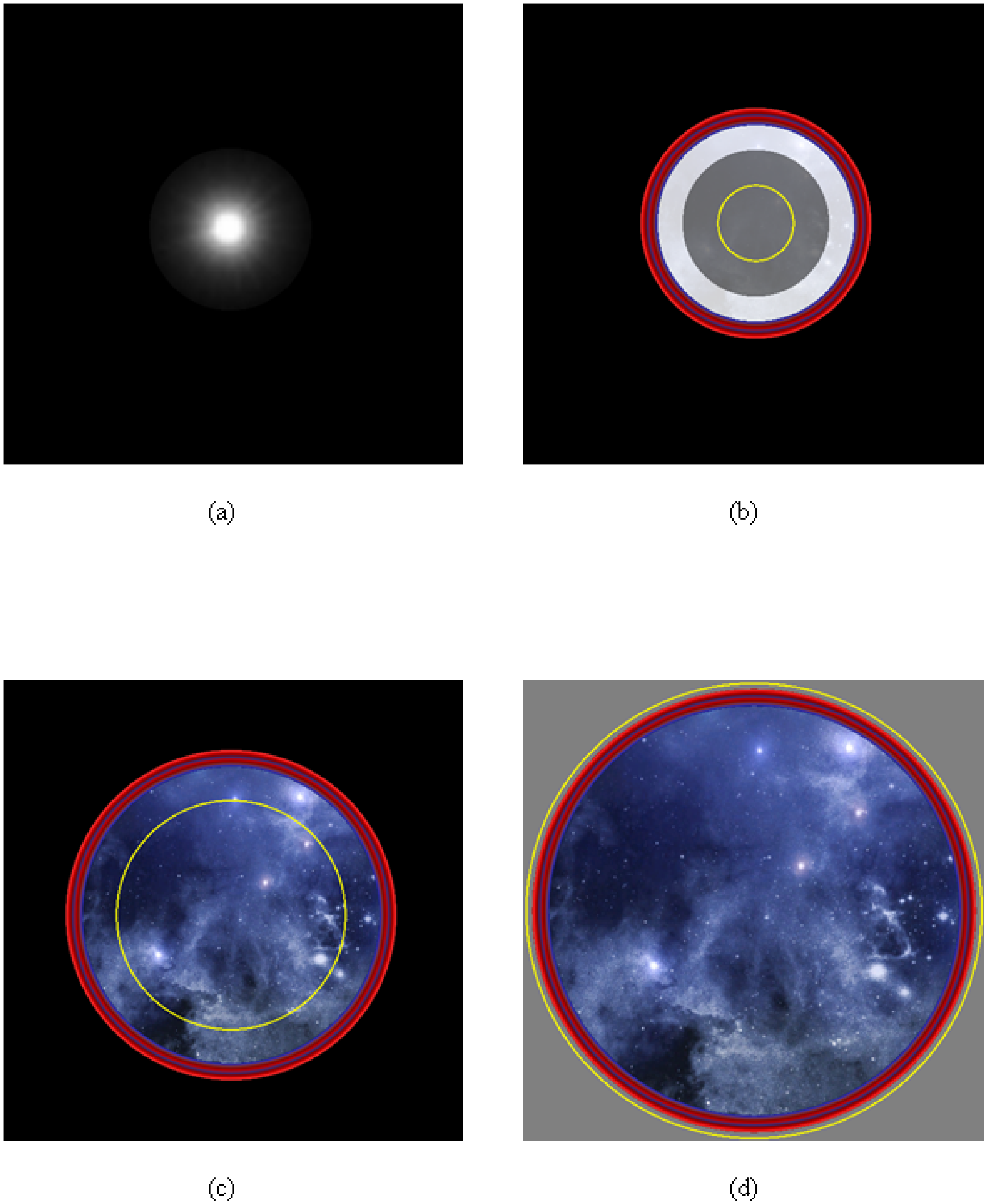


Figure 1. Stages of the Big Bang (a) The beginning (b) Early stage inside the black hole (c) Late stage inside the black hole (d) Last stage outside the black hole

outside the Black Hole, the equation of state $p = \frac{1}{3}\rho$ of early Big Bang physics, plays a special role in the equations, and for this equation of state alone, the behavior of the shock wave at the instant of the Big Bang is distinguished. The entropy condition, (that the density and pressure be larger on the side that receives the mass flux), breaks the time symmetry of the Einstein equations, implies that the shock wave is compressive, and leads to the determination of a unique solution—the FRW metric expanding outward behind a shock wave emanating from a White Hole is entropy satisfying, while its time reversal, the FRW metric contracting into a Black hole, is entropy violating.

Thus, these exact shock wave solutions give the global dynamics of strong gravitational fields in an exact solution, the dynamics is qualitatively different from the dynamics of solutions when the pressure $p \equiv 0$, (c.f. [13]), and the solution suggests a Big Bang cosmological model in which the expanding universe is bounded throughout its expansion. But these solutions are only rough qualitative models because the equation of state on the FRW side of the shock takes the simplified form $p = \sigma\rho$ for $\sigma = constant$, and the equation of state on the TOV side is determined by the equations, and therefore cannot be imposed. For more general equations of state, other waves, (e.g. rarefaction waves), would need to be present to meet the conservation constraint, and thereby mediate the transition across the shock wave. Such transitional waves would be pretty much impossible to model in an exact solution. But the bounds on the pressure in these models imply that the equations of state are qualitatively reasonable, and qualitative phenomena in general relativity, like the stability limits in stars, are rather insensitive to the fine details of a given equation of state. Moreover, since the qualitative features of the solutions are the same for all values of $\sigma$, and for all values of the total mass of the explosion, we expect that these solutions will capture the gross dynamics arising when more accurate equations of state are imposed.

Thus our attempt to incorporate a shock wave beyond one Hubble length has led to unexpected and interesting connections between Big Bang Cosmology and black holes, and we suggest that general relativity pretty much forces the qualitative behavior we see here into any reasonable model that assumes the spacetime is simply connected, close to spherically symmetric, and relaxes the assumption in the standard model that the Big Bang was an explosion of infinite total mass. Moreover, the models imply the existence of a wave out beyond the galaxies that could in principle be observable.

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