The “Big Wave” theory for Dark Energy

Blake Temple

(Joint work with Joel Smoller)

Abstract. We explore the author’s recent proposal that the anomalous acceleration of the galaxies might be due to the displacement of nearby galaxies by a wave that propagated during the radiation phase of the Big Bang.

Mathematics Subject Classification (2010). Primary 83B05; Secondary 35L65.

Keywords. Anomalous acceleration, self-similar waves, conservation laws.

1. Introduction

By obtaining a linear relation between the recessional velocities of distant galaxies (redshift) and luminosity (distance), the American astronomer Edwin Hubble showed in 1927 that the universe is expanding. This confirmed the so-called *standard model of cosmology*, that the universe, on the largest scale, is evolving according to a Friedmann-Robertson-Walker (FRW) spacetime. The starting assumption in this model is the *Cosmological Principle*—that on the largest scale, we are not in a special place in the universe—that, in the words of Robertson and Walker, the universe is *homogeneous* and *isotropic* about every point like the FRW spacetime. In 1998, more accurate measurements of the recessional velocity of distant galaxies based on new Type 1a supernova data made the surprising discovery that the universe was actually accelerating relative to the standard model. This is referred to as the *anomalous acceleration of the galaxies.* The only way to preserve the FRW framework and the Cosmological Principle is to modify the Einstein equations by adding an extra term called the *cosmological constant. Dark Energy*, the physical interpretation of the cosmological constant, is then an unknown source of anti-gravitation that, for the model to be correct, must account for some 70 percent of the energy density of the universe.

Supported by NSF Applied Mathematics Grant Number DMS-070-7532.

In [14] the authors introduced a family of self-similar expanding wave solutions of the Einstein equations of General Relativity (GR) that contain the standard model during the radiation phase of the Big Bang. Here I discuss our cosmological interpretation of this family, and explore the possibility that waves in the family might account for the anomalous acceleration of the galaxies without the cosmological constant or Dark Energy (see [14, 16] for details). In a nutshell, our premise is that the Einstein equations of GR during the radiation phase form a highly nonlinear system of wave equations that support the propagation of waves, and [14] is the culmination of our program to discover waves that perturb the uniform background Friedmann universe (the setting for the standard model of cosmology), something like water waves perturb the surface of a still pond. I also use this as a vehicle to record our unpublished *Answers to reporter's questions* which appeared on the author's website the week our PNAS paper [14] appeared, August 17, 2009.

In Einstein's theory of General Relativity, gravitational *forces* turn out to be just anomalies of spacetime *curvature*, and the propagation of curvature through spacetime is governed by the *Einstein equations.* The Einstein equations during the radiation phase (when the equation of state simplifies to $p = \rho c^2/3$) form a highly nonlinear system of conservation laws that support the propagation of waves, including compressive shock waves and self-similar expansion waves. Yet since the 1930s, the modern theory of cosmology has been based on the starting assumption of the Copernican Principle, which restricts the whole theory to the Friedmann spacetimes, a special class of solutions of the Einstein equations which describe a uniform three-space of constant curvature and constant density evolving in time. Our approach has been to look for general-relativistic waves that could perturb a uniform Friedmann background. The GR self-similar expanding waves in the family derived in [14] satisfy two important conditions: they perturb the standard model of cosmology, and they are the kind of waves that more complicated solutions should settle down to according to the quantitative theories of Lax and Glimm on how solutions of conservation laws decay in time to self-similar wave patterns. The great accomplishment of Lax and Glimm was to explain and quantify how *entropy*, *shock-wave dissipation* and *time-irreversibility* (concepts that originally were understood only in the context of ideal gases) could be given meaning in general systems of *conservation laws*, a setting much more general than gas dynamics. (This viewpoint is well expressed in the celebrated works [10, 5, 6].) The conclusion: Shock-waves introduce dissipation and increase of entropy into the dynamics of solutions, and this provides a mechanism by which complicated solutions can settle down to orderly self-similar wave patterns, *even when dissipative terms are neglected in the formulation of the equations.* A rock thrown into a pond demonstrates how the mechanism can transform a chaotic "plunk" into a series of orderly outgoing self-similar waves moments later. As a result, our new construction of a family of GR self-similar waves that apply when this decay mechanism should be in place received a good deal of media attention when it came out in PNAS, August 2009. (A sampling of press releases and articles can be found on my homepage.[1]

At the value of the *acceleration parameter* $a = 1$ (the free parameter in our family of self-similar solutions), the solution reduces exactly to the critical FRW spacetime of the standard model with pure radiation sources, and solutions look remarkably similar to FRW when $a \neq 1$. When $a \neq 1$, we prove that the spacetimes in the family are distinct from all the other non-critical FRW spacetimes, and hence it follows that the critical FRW during the radiation phase is characterized as the unique spacetime lying at the intersection of these two one-parameter families. Since adjustment of the free parameter a speeds up or slows down the expansion rate relative to the standard model, we argue they can account for the leading-order quadratic correction to redshift vs luminosity observed in the supernova data, without the need for Dark Energy. I first proposed the idea that the anomalous acceleration might be accounted for by a wave in the talk *Numerical Shockwave Cosmology*, New Orleans, January 2007,[2] and set out to simulate such a wave numerically. While attempting to set up the numerical simulation, we discovered that the standard model during the radiation phase admits a coordinate system (Standard Schwarzschild Coordinates (SSC)) in which the Friedmann spacetime is *self-similar*. That is, it took the form of a non-interacting time-asymptotic wave pattern according to the theory of Lax and Glimm. This was the key. Once we found this, we guessed that the Einstein equations in these coordinates must close to form a new system of ODEs in the same self-similar variable. After a struggle, we derived this system of equations, and showed that the standard model was one point in a family of solutions parameterized by four initial conditions. Symmetry and regularity at the center then reduced the four-parameter family to an implicitly defined one-parameter family, one value of which gives the critical Friedmann spacetime of the standard model during the radiation phase of the Big Bang. Our idea then: an expansion wave that formed during the radiation epoch, when the Einstein equations obey a highly nonlinear system of conservation laws for which we must expect self-similar non-interacting waves to be the end state of local fluctuations, could account for the anomalous acceleration of the galaxies without Dark Energy. Since we have explicit formulas for such waves, it is a verifiable proposition.

2. Statement of results

In this section we state three theorems which summarize our results in [14, 16]. (Unbarred coordinates (t, r) refer to FRW co-moving coordinates, and barred coordinates $(\bar{t}, \bar{r})$ refer to (SSC).)

---

[1] see *Media Articles* on my homepage `http://www.math.ucdavis.edu/~temple/`

[2] the fourth entry under *Conference/Seminar Talks* on my homepage

Theorem 2.1. *Assume* $p=\frac{1}{3}\rho c^2$, $k=0$ *and* $R(t)=\sqrt{t}$. *Then the FRW metric*

$$ds^2=-dt^2+R(t)^2dr^2+\bar{r}^2d\Omega^2,$$

under the change of coordinates

$$\bar{t} = \psi_0\left\{1+\left[\frac{R(t)r}{2t}\right]^2\right\}t, \tag{2.1}$$

$$\bar{r} = R(t)r, \tag{2.2}$$

transforms to the SSC-metric

$$ds^2=-\frac{d\bar{t}^2}{\psi_0^2\left(1-v^2(\xi)\right)}+\frac{d\bar{r}^2}{1-v^2(\xi)}+\bar{r}^2d\Omega^2, \tag{2.3}$$

where

$$v=\frac{1}{\sqrt{AB}}\frac{\bar{u}^1}{\bar{u}^0} \tag{2.4}$$

is the SSC velocity, which also satisfies

$$v=\frac{\zeta}{2}, \tag{2.5}$$

$$\psi_0\xi=\frac{2v}{1+v^2}. \tag{2.6}$$

Theorem 2.2. *Let* ξ *denote the self-similarity variable*

$$\xi=\frac{\bar{r}}{\bar{t}}, \tag{2.7}$$

and let

$$G=\frac{\xi}{\sqrt{AB}}. \tag{2.8}$$

Assume that $A(\xi)$, $G(\xi)$ *and* $v(\xi)$ *solve the ODEs*

$$\xi A_\xi = -\left[\frac{4(1-A)v}{(3+v^2)G-4v}\right] \tag{2.9}$$

$$\xi G_\xi = -G\left\{\left(\frac{1-A}{A}\right)\frac{2(1+v^2)G-4v}{(3+v^2)G-4v}-1\right\} \tag{2.10}$$

$$\xi v_\xi = -\left(\frac{1-v^2}{2\left\{\cdot\right\}_D}\right)\left\{(3+v^2)G-4v+\frac{4\left(\frac{1-A}{A}\right)\left\{\cdot\right\}_N}{(3+v^2)G-4v}\right\}, \tag{2.11}$$

where

$$\left\{\cdot\right\}_N = \left\{-2v^2+2(3-v^2)vG-(3-v^4)G^2\right\} \tag{2.12}$$

$$\left\{\cdot\right\}_D = \left\{(3v^2-1)-4vG+(3-v^2)G^2\right\}, \tag{2.13}$$

and define the density by

$$\kappa\rho=\frac{3(1-v^2)(1-A)G}{(3+v^2)G-4v}\frac{1}{\bar{r}^2}. \tag{2.14}$$

Then the metric

$$ds^2 = -B(\xi)d\bar{t}^2 + \frac{1}{A(\xi)}d\bar{r}^2 + \bar{r}^2 d\Omega^2 \tag{2.15}$$

solves the Einstein-Euler equations $G = \kappa T$ *with velocity* $v = v(\xi)$ *and equation of state* $p = \frac{1}{3}\rho c^2$*. In particular, the FRW metric* (2.3) *solves equations* (2.9)–(2.11)*.*

Note that it is not evident from the FRW metric in standard co-moving coordinates that self-similar variables even exist, and if they do exist, by what ansatz one should extend the metric in those variables to obtain nearby self-similar solutions that solve the Einstein equations exactly. The main point is that our coordinate mapping to SSC explicitly identifies the self-similar variables as well as the metric ansatz that together accomplish such an extension of the metric.

In [14, 16] we prove that the three-parameter family (2.9)–(2.11) (parameterized by three initial conditions) reduces to an (implicitly defined) one-parameter family by removing time-scaling invariance and imposing regularity at the center. The remaining parameter a changes the expansion rate of the spacetimes in the family, and thus we call it the *acceleration parameter*. Transforming back to (approximate) co-moving coordinates, the resulting one-parameter family of metrics is amenable to the calculation of a redshift vs luminosity relation, to third order in the redshift factor z, leading to the following theorem which applies during the radiation phase of the expansion, cf. [14, 16]:

Theorem 2.3. *The redshift vs luminosity relation, as measured by an observer positioned at the center of the expanding wave spacetimes (metrics of form* (2.15)*), is given up to fourth order in redshift factor* z *by*

$$d_\ell = 2ct\left\{z + \frac{a^2-1}{2}z^2 + \frac{(a^2-1)(3a^2+5)}{6}z^3 + O(1)|a-1|z^4\right\}, \tag{2.16}$$

where d_ℓ *is luminosity distance,* ct *is invariant time since the Big Bang, and* a *is the acceleration parameter that distinguishes expanding waves in the family.*

When $a = 1$, (2.16) reduces to the correct linear relation of the standard model, [8]. Assuming redshift vs luminosity evolves continuously in time, it follows that the leading-order part of any (small) anomalous correction to the redshift vs luminosity relation of the standard model observed *after* the radiation phase could be accounted for by suitable adjustment of parameter a.

3. Discussion

These results suggest an interpretation that we might call a *conservation law* explanation of the anomalous acceleration of the galaxies. That is, the theory of Lax and Glimm explains how highly interactive oscillatory solutions of conservation laws will decay in time to non-interacting waves (shock waves and expansion waves), by the mechanisms of wave interaction and shock-wave dissipation. The subtle point is that even though dissipation terms are neglected in the formulation of the equations, there is a canonical dissipation and consequent loss of information due to the *nonlinearities*, and this can be modeled by shock-wave interactions that drive solutions to non-interacting wave patterns. Since the one fact most certain about the standard model is that our universe arose from an earlier hot dense epoch in which all sources of energy were in the form of radiation, and since it is approximately uniform on the largest scale but highly oscillatory on smaller scales[3], one might reasonably conjecture that decay to a non-interacting expanding wave occurred during the radiation phase of the standard model, via the highly nonlinear evolution driven by the large sound speed, and correspondingly large modulus of *genuine nonlinearity*[4], present when $p = \rho c^2/3$, cf. [11]. Our analysis has shown that FRW is just one point in a family of non-interacting, self-similar expansion waves, and as a result we conclude that some further explanation is required as to why, on some length scale, decay during the radiation phase of the standard model would not proceed to a member of the family satisfying $a \neq 1$. If decay to $a \neq 1$ did occur, then the galaxies that formed from matter at the end of the radiation phase (some $379,000$ years after the Big Bang) would be displaced from their anticipated positions in the standard model at present time, and this displacement would lead to a modification of the observed redshift vs luminosity relation. In short, the displacement of the fluid particles (i.e., the displacement of the co-moving frames in the radiation field) by the wave during the radiation epoch leads to a displacement of the galaxies at a later time. In principle such a mechanism could account for the anomalous acceleration of the galaxies as observed in the supernova data. Of course, if $a \neq 1$, then the spacetime within the expansion wave has a center, and this would violate the so-called *Copernican Principle*, a simplifying assumption generally accepted in cosmology, at least on the scale of the wave (cf. the discussions in [17] and [1]). Moreover, if our Milky Way galaxy did not lie within some threshold of the center of expansion, the expanding wave theory would imply unobserved angular variations in the expansion rate. In fact, all of these observational issues have already been discussed recently in [2, 1, 3] (and references therein), which explore the possibility that the anomalous acceleration of the galaxies might be due to a local *void* or under-density of galaxies in the vicinity of the Milky Way.[5] Our proposal then is

[3]In the standard model, the universe is approximated by uniform density on a scale of a billion light years or so, about a tenth of the radius of the visible universe, [18]. The stars, galaxies and clusters of galaxies are then evidence of large oscillations on smaller scales.
[4]Again, *genuine nonlinearity* is, in the sense of Lax, a measure of the magnitude of nonlinear compression that drives decay, cf. [10].
[5]The size of the center, consistent with the angular dependence that has been observed in the actual supernova and microwave data, has been estimated to be about 15 megaparsecs, approximately the distance between clusters of galaxies, roughly 1/200 the distance across the visible universe, cf. [1, 2, 3].

that the one-parameter family of general-relativistic self-similar expansion waves derived here is a family of possible end-states that could result after dissipation by wave interactions during the radiation phase of the standard model is completed, and such waves could thereby account for the appearance of a local under-density of galaxies at a later time.

In any case, the expanding wave theory is testable. For a first test, we propose next to evolve the quadratic and cubic corrections to redshift vs luminosity recorded here in relation (2.16), valid at the end of the radiation phase, up through the $p \approx 0$ stage to present time in the standard model, to obtain the present-time values of the quadratic and cubic corrections to redshift vs luminisity implied by the expanding waves, as a function of the acceleration parameter a. Once accomplished, we can look for a best fit value of a via comparison of the quadratic correction at present time to the quadratic correction observed in the supernova data, leaving the third-order correction at present time as a prediction of the theory. That is, in principle, the predicted third-order correction term could be used to distinguish the expanding wave theory from other theories (such as Dark Energy) by the degree to which they match an accurate plot of redshift vs luminosity from the supernove data (a topic of the authors' current research). The idea that the anomalous acceleration might be accounted for by a local under-density in a neighborhood of our galaxy was expounded in the recent papers [2, 3]. Our results here might then give an accounting for the source of such an under-density.

The expanding wave theory could in principle give an explanation for the observed anomalous acceleration of the galaxies within classical General Relativity, with classical sources. In the expanding wave theory, the so-called anomalous acceleration is not an acceleration at all, but is a correction to the standard model due to the fact that we are looking outward into an expansion wave. The one-parameter family of non-interacting, self-similar, general-relativistic expansion waves derived here contains all possible end-states that could result by wave interaction and dissipation due to nonlinearities back when the universe was filled with pure radiation sources. And when $a \neq 1$, they introduce an anomalous acceleration into the standard model of cosmology. Unlike the theory of Dark Energy, this provides a possible explanation for the anomalous acceleration of the galaxies that is not *ad hoc* in the sense that it is derivable exactly from physical principles and a mathematically rigorous theory of general-relativistic expansion waves. In particular, this explanation does not require the *ad hoc* assumption of a universe filled with an as yet unobserved form of energy with anti-gravitational properties (the standard physical interpretation of the cosmological constant) in order to fit the data.

In summary, these new general-relativistic expanding waves provide a new paradigm to test against the standard model. Even if they do not in the end explain the anomalous acceleration of the galaxies, one has to believe they are present and propagating on some scale, and their presence represents an instability in the standard model in the sense that an explanation is required as to why small-scale oscillations have to settle down to large-scale $a = 1$ expansions instead of $a \neq 1$ expansions (either locally or globally) during the radiation phase of the Big Bang.

We now use this proceedings to record the *Answers to reporter's questions* which appeared on our websites shortly after our PNAS paper came out in August 2009.

4. Answers to reporter's questions: Blake Temple and Joel Smoller, August 17, 2009.

To Begin: Let us say at the start that what is definitive about our work is the construction of a new one-parameter family of exact, self-similar expanding wave solutions to Einstein's equations of General Relativity. They apply during the radiation phase of the Big Bang, and approximate the standard model of cosmology arbitrarily well. For this we have complete mathematical arguments that are not controvertible. Our intuitions that led us to these, and their physical significance to the anomalous acceleration problem, are based on lessons learned from the mathematical theory of nonlinear conservation laws, and only this interpretation is subject to debate.

1) *Could you explain—in simple terms—what an expanding wave solution is and what other phenomena in nature can be explained through this mathematics?*

To best understand what an expanding wave is, imagine a stone thrown into a pond, making a splash as it hits the water. The initial "plunk" at the start creates chaotic waves that break every which way, but after a short time the whole disturbance settles down into orderly concentric circles of waves that radiate outward from the center—think of the resulting final sequence of waves as the "expanding wave". In fact, it is the initial breaking of waves that dissipates away all of the disorganized motion, until all that is left is the orderly expansion of waves. For us, the initial "plunk" of the stone is the chaotic Big Bang at the start of the radiation phase, and the expansion wave is the orderly expansion that emerges at the end of the radiation phase. What we have found is that the standard model of cosmology is not the only expanding wave that could emerge from the initial "plunk". In fact, we constructed a whole family of possible expanding waves that could emerge; and we argue that which one would emerge depends delicately on the nature of the chaos in the initial "plunk". That is, one expanding wave in the family is equally likely to emerge as another. Our family depends on a freely assignable number a which we call the *acceleration parameter*, such that if we pick $a = 1$, then we get the standard model of cosmology, but if $a > 1$ we get an expanding wave that looks a lot like the standard model, but expands faster, and if $a < 1$, then it expands slower. So an "anomalous acceleration" would result if $a > 1$.

Summary: By "expanding wave" we mean a wave that expands outward in a "self-similar" orderly way in the sense that at each time the wave looks like it did at an earlier time, but more "spread out". The importance of an expanding wave is that it is the end state of a chaotic disturbance because it is what remains after all the complicated breaking of waves is over... one part of the expanding solution no longer affects the other parts. Our thesis, then, is that we can account for the anomalous acceleration of the galaxies without Dark Energy by taking $a > 1$.

2) *Could you explain how and why you decided to apply expanding wave solutions to this particular issue?*

We (Temple) got the idea that the anomalous acceleration of the galaxies might be explained by a secondary expansion wave reflected backward from the shock wave in our earlier construction of a shock wave in the standard model of cosmology, and proposed to numerically simulate such a wave. Temple got this idea while giving a public lecture to the National Academy of Sciences in Bangalore India, in 2006. We set out together to simulate this wave while Temple was Gehring Professor in Ann Arbor in 2007, and in setting up the simulation, we subsequently discovered exact formulas for a family of such waves, without the need for the shock wave model.

3) *Do you think this provides the strongest evidence yet that Dark Energy is a redundant idea?*

At this stage we personally feel that this gives the most plausible explanation for the anomalous acceleration of the galaxies that does not invoke Dark Energy. Since we don't believe in "Dark Energy"... [more detail in (12) below].

We emphasize that our model implies a verifiable prediction, so it remains to be seen whether the model fits the red-shift vs luminosity data better than the Dark Energy theory. (We are working on this now.)

4) *Is this the first time that expanding wave solutions of the Einstein equations have been realized?*

As far as we know, this is the first time a family of self-similar expanding wave solutions of the Einstein equations has been constructed for the radiation phase of the Big Bang, such that the members of the family can approximate the standard model of cosmology arbitrarily well. Our main point is *not* that we have self-similar expanding waves, but that we have self-similar expanding waves during the radiation phase when (1) decay to such waves is possible because $p \neq 0$, and (2) they are close to the standard model. We are not so interested in self-similar waves when $p = 0$ because we see no reason to believe that self-similar waves during the time when $p = \rho c^2/3 \neq 0$ will evolve into exact self-similar waves in the present era when $p = 0$. That is, they should evolve into some sort of expanding spacetime when $p = 0$, but not a pure (self-similar) expansion wave.

5) *How did you reach the assumption that* $p = [\rho][c]^2/3$, *a wise one?*

We are mathematicians, and in the last several decades, a theory for how highly nonlinear equations can decay to self-similar waves was worked out by mathematicians, starting with fundamental work of Peter Lax and Jim Glimm. The theory was worked out for model equations much simpler than the Einstein equations. We realized that only during the radiation phase of the expansion were the equations "sufficiently nonlinear" to expect sufficient breaking of waves at the start to create enough dissipation to drive a chaotic disorganized disturbance into an orderly self-similar expansion wave at the end. The subtle point is that even though no mechanisms for dissipation are put into the model, the nonlinearities alone can cause massive dissipation via the breaking of waves that would drive a chaotic disturbance into an orderly expansion wave.

6) *How does your suggestion—that the observed anomalous acceleration of the galaxies could be due to our view into an expansion wave—compare with an idea that I heard Subir Sarkar describe recently: that the Earth could be in a void that is expanding faster than the outer parts of the universe?*

We became aware of this work in the fall of '08, and forwarded our preprints. Our view here is that after the radiation phase is over, and the pressure drops to zero, there is no longer any nonlinear mechanism that can cause the breaking of waves that can cause dissipation into an expansion wave. Thus during the recent $p = 0$ epoch (after some 300,000 years after the Big Bang), you might model the evolution of the remnants of such an expanding wave or under-density (in their terms a local "void"), but there is no mechanism in the $p = 0$ phase to explain the constraints under which such a void could form. (When $p = 0$, everything is in "freefall", and there can be no breaking of waves.) The expanding wave theory we present provides a possible quantitative explanation for the formation of such a void.

7) *How do you intend to develop your research from here?*

Our present paper demonstrates that there is some choice of the number a (we proved it exists, but still do not know its precise value) such that the member of our family of expanding waves corresponding to that value of the acceleration parameter a will account for the leading-order correction of the anomalous acceleration. That is, it can account for how the plot of redshift vs luminosity of the galaxies curves away from a straight line at the center. But once the correct value of a is determined exactly, that value will give a prediction of how the plot should change beyond the first breaking of the curve. (There are no more free parameters to adjust!) We are currently working on finding that exact value of a consistent with the observed anomalous acceleration, so that from this we can calculate the next-order correction it predicts, all with the goal of comparing the expanding wave prediction to the observed redshift vs luminosity plot, to see if it does better than the prediction of Dark Energy.

8) *What is your view on the relevance of the Copernican Principle to these new expanding waves?*

These self-similar expanding waves represent possible end states of the expansion of the Big Bang that we propose could emerge at the end of the radiation phase when there exists a mechanism for their formation. We imagine that decay to such an expanding wave could have occurred locally in the vicinity of the Earth, over some length scale, but we can only conjecture as to what length scale that might be—the wave could extend out to some fraction of the distance across the visible universe or it could extend even beyond, we cannot say, but to explain the anomalous acceleration the Earth must lie within some proximity of the center. That is, for the $a > 1$ wave to account for the anomalous acceleration observed in the galaxies, we would have to lie in some proximity of the center of such a wave to be consistent with no observed angular dependence in the redshift vs luminosity plots. (The void theory has the same implication.)

Now one might argue that our expanding waves violate the so-called *Copernican* or *Cosmological Principle* which states that *on the largest length scale* the universe looks the same everywhere. This has been a simplifying assumption taken in cosmology since the mid thirties when Howard Robertson and Geoffrey Walker proved that the Friedmann spacetimes of the standard model (constructed by Alexander Friedmann a decade earlier) are the unique spacetimes that are spatially *homogeneous* and *isotropic* about every point—a technical way of saying there is no special place in the universe. The introduction of Dark Energy via the cosmological constant is the only way to preserve the Copernican Principle *and* account for the anomalous acceleration on the largest scale, *everywhere.* The stars, galaxies and clusters of galaxies are evidence of small-scale variations that violate the Copernican Principle on smaller length scales. We are arguing that there could be an even larger length scale than the clusters of galaxies on which local decay to one of our expanding waves has occurred, and we happen to be near the center of one. This would violate the Copernican Principle if these expanding waves describe the entire universe—but our results allow for the possibility that on a scale even larger than the scale of the expanding waves, the universe may look everywhere the same like the standard model. Thus our view is that the Copernican Principle is really a moot issue here. But it does beg the question as to how big the effective center can be for the value of a that accounts for the anomalous acceleration. This is a problem we hope to address in the future.

Another way to look at this is, if you believe there is no cosmological constant or Dark Energy [see (12) below], then the anomalous acceleration may really be the first definitive evidence that in fact, by accident, we just happen to lie near the center of a great expansion wave of cosmic dimensions. We believe our work at this stage gives strong support for this possibility.

9) *How large would the displacement of matter caused by the expanding wave be, and how far out would it extend?*

For our model, the magnitude of the displacement depends on the value of the acceleration parameter a. It can be very large or very small, and we argue that somewhere in between it can be right on for the first breaking of the observed redshift vs luminosity curve near the center. To meet the observations, it has to displace the position of a distant galaxy the right amount to displace the straight line redshift vs luminosity plot of the standard model into the curved graph observed. In their article referenced in our paper (exposition of this appeared as the cover article in Scientific American a few months ago), Clifton and Ferreira quote that the bubble of under-density observed today should extend out to about one billion lightyears, about a tenth of the distance across the visible universe, and the size of the center consistent with no angular variation is about 15 megaparsecs, about 50 million lightyears, and this is approximately the distance between clusters of galaxies, a distance about 1/200 across the visible universe.[6]

10) *How do the spacetimes associated with the expanding waves compare to the spacetime of the standard model of cosmology?*

Interestingly, we prove that the spacetimes associated with the expanding waves when $a \neq 1$ actually have properties surprisingly similar to the standard model $a = 1$. Firstly, the expanding spacetimes ($a \neq 1$) look more and more like the standard model $a = 1$ as you approach the center of expansion. (That is why you have to go far out to see an anomalous acceleration.)

---

[6] The following back-of-the-envelope calculation provides a ballpark estimate for what we might expect the extent of the remnants of one of our expanding waves might be today. Our thesis is that the self-similar expanding waves that can exist during the pure radiation phase of the standard model can emerge at the end of the radiation phase by the dissipation created by the strong nonlinearities. Now matter becomes transparent with radiation at about 300,000 years after the Big Bang, so we might estimate that our wave should have emerged by about $t_{\text{endrad}} \approx 10^5$ years after the Big Bang. At this time, the distance of light-travel since the Big Bang is about 10^5 lightyears. Since the sound speed $c/\sqrt{3} \approx .58c$ during the radiation phase is comparable to the speed of light, we could estimate that dissipation that drives decay to the expanding wave might reasonably be operating over a scale of 10^5 lightyears by the end of the radiation phase. Now in the $p = 0$ expansion that follows the radiation phase, the scale factor (that gives the expansion rate) evolves like

$$R(t) = t^{2/3},$$

so a distance of 10^5 lightyears at $t = t_{\text{endrad}}$ years will expand to a length L at present time $t_{\text{present}} \approx 10^{10}$ years by a factor of

$$\frac{R(t_{\text{present}})}{R(t_{\text{endrad}})} \approx \frac{(10^{10})^{2/3}}{(10^5)^{2/3}} = 10^{4.7} \geq 5 \times 10^4.$$

It follows then that we might expect the scale of the wave at present time to extend over a distance of about

$$L = 5 \times 10^5 \times 10^4 = 5 \times 10^9 \ \text{lightyears}.$$

This is a third to a fifth of the distance across the visible universe, and agrees with the extent of the under-density void region quoted in the Clifton-Ferreira paper, with room to spare.

Moreover, out to a great distance from the center, say out to about 1/3 to 1/2 the distance across the visible universe, (where the anomalous acceleration is apparent), we prove that (to within negligible errors) there is a time coordinate t such that the 3-space at each fixed t has zero curvature, just like the standard model of cosmology, and observers fixed in time or at a fixed distance from the center will measure distances and times exactly the same as in a Friedmann universe, the spacetime of the standard model of cosmology. In technical terms, only line elements changing in space *and* time will measure dilation of distances and times relative to the standard model. This suggests that it would be easy to mistake one of these expanding waves for the Friedmann spacetime itself until you did a measurement of redshift vs luminosity far out where the differences are highly apparent (that is, you measured the anomalous acceleration).

11) *Your expanding wave theory is more complicated than a universe filled with Dark Energy, and we have to take into account the Occam's razor principle. What do you think about this assertion?*

To quote Wikipedia, Occam's razor states: "The explanation of any phenomenon should make as few assumptions as possible, eliminating those that make no difference in the observable predictions of the explanatory hypothesis or theory."

We could say that our theory does not require the extra hypothesis of Dark Energy or a cosmological constant to explain the anomalous acceleration. Since there is no obvious reason why an expansion wave with one value of a over another would come out locally at any given location at the end of the radiation phase, and since we don't need Dark Energy in the expanding wave explanation, we could argue that the expanding wave explanation of the anomalous acceleration is simpler than Dark Energy. But a better answer is that our theory has an observable prediction, and only experiments, not the 14th-century principle of Occam, can resolve the physics. Occam's razor will have nothing whatsoever to say about whether we are, or are not, near the center of a cosmic expansion wave.

12) *If, as you suggest, Dark Energy doesn't exist, what is the ingredient of 75% of the mass-energy in our universe?*

In short, nothing is required to replace it. The term "anomalous acceleration" of the galaxies begs the question "acceleration relative to what?". The answer is that the anomalous acceleration of the galaxies is an acceleration relative to the prediction of the standard model of cosmology. In the expanding wave theory, we prove that there is no "acceleration" because the anomalous acceleration can be accounted for in redshift vs luminosity by the fact that the galaxies in the expanding wave are displaced from their anticipated position in the standard model. So the expanding wave theory requires only classical sources of mass-energy for the Einstein equations.

13) *If Dark Energy doesn't exist, it would be just an invention. What do you think about Dark Energy theory?*

Keep in mind that Einstein's equations have been confirmed without the need for the cosmological constant or Dark Energy, in every physical setting except in cosmology.

Dark Energy is the physical interpretation of the cosmological constant. The cosmological constant is a source term with a free parameter (similar to but different from our a) that can be added to the original Einstein equations and still preserve the frame independence, the "general relativity" if you will, of Einstein's equations. Einstein's equations express that mass-energy is the source of spacetime curvature. So if you interpret the cosmological constant as the effect of some exotic mass-energy, then you get Dark Energy. For the value of the cosmological constant required to fit the anomalous acceleration observed in the redshift vs luminosity data, this Dark Energy must account for some 73 percent of the mass-energy of the universe, and it has to have the physical property that it *anti-gravitates*—that is, it gravitationally repels instead of attracts. Since no one has ever observed anything that has this property (it would not fall to Earth like an apple, it would fly up like a balloon), it seems rather suspect that such mass-energy could possibly exist. If it does exist, then it also is not like any other mass-energy in that the density of it stays constant, stuck there at the same value forever, even as the universe expands and spreads all the other mass-energy out over larger and larger scales—and there is no principle that explains why it has the value it has.[7] On the other hand, if you put the cosmological constant on the other side of the equation with the curvature then there is always some (albeit very small) baseline curvature permeating spacetime, and the zero-curvature spacetime is no longer possible; that is, the empty-space Minkowski spacetime of Special Relativity no longer solves the equations. So when the cosmological constant is over on the curvature side of Einstein's equation, the equations no longer express the physical principle that led Einstein to discover them in the first place—that mass-energy should be the sole source of spacetime curvature.

Einstein put the cosmological constant into his equations shortly after he discovered them in 1915, because this was the only way he could get the possibility of a static universe. (*Anti-gravity* holds the static universe up!) After Hubble proved that the universe was expanding in 1929, Einstein took back the cosmological constant, declaring it was the greatest blunder of his career, as he could have predicted the expansion ahead of time without it. At the time, taking out the cosmological constant was interpreted as a great victory for General Relativity. Since then, cosmologists have become more comfortable putting the cosmological constant back in. There are many respected scientists who see no problem with Dark Energy.

---

[7] In the expanding wave theory, the principle for determining a is that all values of a near $a = 1$ should be (roughly) equally likely to appear, and one of them did...

14) *How does the coincidence in the value of the cosmological constant in the Dark Energy theory compare to the coincidence that the Milky Way must lie near a local center of expansion in the expanding wave theory?*

The Dark Energy explanation of the anomalous acceleration of the galaxies requires a value of the cosmological constant that accounts for some 73 percent of the mass-energy of the universe. That is, to correct for the anomalous acceleration in the supernova data, you need a value of the cosmological constant that is just three times the energy-density of the rest of the mass-energy of the universe. Now there is no principle that determines the value of the cosmological constant ahead of time, so its value could apriori be anything. Thus it must be viewed as a great coincidence that it just happens to be so close to the value of the energy density of the rest of the mass-energy of the universe. (Keep in mind that the energy-density of all the classical sources decreases as the universe spreads out, while the cosmological constant stays *constant.*) So why does the value of the cosmological constant come out so close to, *just 3 times*, the value of the rest of the mass-energy of the universe, instead of 10^{10} larger or 10^{-10} smaller? This raises a very suspicious possibility. Since the magnitude of the sources sets the scale for the overall *oomph* of the solution, when you need to adjust the equations by an amount on the order of the sources present in order to fit the data, that smacks of the likelihood that you are really just adding corrections to the wrong underlying solution. So to us it looks like the coincidence in the value of the cosmological constant in the Dark Energy theory may well be greater than the coincidence that we lie near a local center of expansion in the expanding wave theory.

In summary: Our view is that the Einstein equations make more physical sense without Dark Energy or the cosmological constant, and Dark Energy is most likely an unphysical *fudge factor*, if you will, introduced into the theory to meet the data. But ultimately, whether Dark Energy or an expanding wave correctly explains the anomalous acceleration of the galaxies can only be decided by experiments, not the Copernican Principle or Occam’s razor.

References

[1] C. Copi, D. Huterer, D.J. Schwarz, G.D. Starkman, *On the large-angle anomalies of the microwave sky*, Mon. Not. R. Astron. Soc. **367**, 79–102 (2006).

[2] T. Clifton, P.G. Ferreira, K. Land, *Living in a void: testing the Copernican principle with distant supernovae*, Phys. Rev. Lett. **101**, 131302 (2008). arXiv:0807.1443v2 [astro-ph].

[3] T. Clifton and P.G. Ferreira, *Does dark energy really exist?*, Sci. Am., April 2009, 48–55.

[4] D. Eardley, *Self-similar spacetimes: geometry and dynamics*, Commun. Math. Phys. **37**, 287–309 (1974).

[5] J. Glimm, *Solutions in the large for nonlinear hyperbolic systems of equations*, Comm. Pure Appl. Math. **18**, 697–715 (1965).

[6] J. Glimm, P.D. Lax, *Decay of solutions of systems of nonlinear hyperbolic conservation laws*, Memoirs Amer. Math Soc. **101** (1970).

[7] J. Groah and B. Temple, *Shock-wave solutions of the Einstein equations: existence and consistency by a locally inertial Glimm scheme*, Memoirs Amer. Math Soc. **172** (2004), no. 813.

[8] O. Gron and S. Hervik, *Einstein's general theory of relativity with modern applications in cosmology*, Springer, 2007.

[9] P. Hartman, *A lemma in the structural stability of ordinary differential equations*, Proc. Amer. Math. Soc. **11**, 610–620 (1960).

[10] P.D. Lax, *Hyperbolic systems of conservation laws, II*, Comm. Pure Appl. Math. **10**, 537–566 (1957).

[11] J. Smoller and B. Temple, *Global solutions of the relativistic Euler equations*, Comm. Math. Phys. **157**, 67–99 (1993).

[12] J. Smoller and B. Temple, *Cosmology, black holes, and shock waves beyond the Hubble length*, Meth. Appl. Anal. **11**, 77–132 (2004).

[13] J. Smoller and B. Temple, *Shock-wave cosmology inside a black hole*, Proc. Nat. Acad. Sci. **100**, 11216–11218 (2003), no. 20.

[14] B. Temple and J. Smoller, *Expanding wave solutions of the Einstein equations that induce an anomalous acceleration into the Standard Model of Cosmology*, Proc. Nat. Acad. Sci. **106**, 14213–14218 (2009), no. 34.

[15] B. Temple and J. Smoller, *Answers to questions posed by reporters: Temple–Smoller GR expanding waves*, August 19, 2009. `http://www.math.ucdavis.edu/~temple/`.

[16] J. Smoller and B. Temple, *General relativistic self-similar waves that induce an anomalous acceleration into the Standard Model of Cosmology*, Memoirs Amer. Math. Soc., to appear.

[17] B. Temple, *Numerical refinement of a finite mass shock-wave cosmology*, AMS National Meeting, Special Session *Numerical Relativity*, New Orleans (2007). `http://www.math.ucdavis.edu/~temple/talks/NumericalShockWaveCosTalk.pdf`.

[18] S. Weinberg, *Gravitation and cosmology: principles and applications of the general theory of relativity*, John Wiley & Sons, 1972.

Blake Temple
Dept of Mathematics
1 Shields Avenue
Davis, CA
USA 95618
e-mail: `temple@math.ucdavis.edu`