AMS/IP Studies in Advanced Mathematics
Volume 51, 2011

A One Parameter Family of Expanding Wave Solutions of the Einstein Equations that induces An Anomalous Acceleration into the Standard Model of Cosmology

Joel Smoller and Blake Temple

ABSTRACT. I discuss joint work with Blake Temple in which we propose that the anomalous acceleration of the galaxies might be due to the displacement of nearby galaxies by a wave that propagated during the radiation phase of the Big Bang. The new result reported here is the calculation of the third order correction to redshift vs luminosity by an application of Etherington's Theorem.

1. Introduction

The anomalous acceleration of the galaxies was first observed in 1998-1999 from accurate measurements of the recessional velocity of distant galaxies based on Type 1a supernova data. The data confirmed that galaxies are receding from us at an accelerated rate relative to the Standard Model of Cosmology based on Friedmann-Robertson-Walker spacetimes (FRW). The current explanation by physicists preserves the FRW framework and the Cosmological Principle by adding a correction term called the *Cosmological Constant* to Einstein's original equations. Dark Energy, the physical interpretation of the cosmological constant, is then an unknown source of anti-gravitation which, to account for the anomalous acceleration, must account for some seventy-three percent of the energy density of the universe.

In this talk I describe authors' recent work in [**17**] and [**20**] in which we derive a one parameter family of self-similar expanding wave solutions of the Einstein equations of General Relativity (GR) that contain the Standard Model during the radiation phase of the Big Bang. I then discuss our cosmological interpretation of this family, and explore the possibility that these self-similar waves might account for the anomalous acceleration of the galaxies without the Cosmological Constant or Dark Energy (see [**20**] for details). Our premise is that the Einstein equations of GR during the radiation phase form a highly nonlinear system of wave equations that support the propagation of waves, and [**17**] is the culmination of our program

Supported by NSF Applied Mathematics Grant Number DMS-060-3754.
Supported by NSF Applied Mathematics Grant Number DMS-070-7532.

to discover waves that perturb the uniform background Friedmann universe, (the setting for the Standard Model of cosmology), something like water waves perturb the surface of a still pond.

In Einstein's theory of general relativity, gravitational *forces* are just anomalies of spacetime *curvature*, and the propagation of curvature through spacetime is governed by the Einstein equations. The Einstein equations during the radiation phase, (when the equation of state simplifies to $p = \rho c^2/3$), form a highly nonlinear system of conservation laws that support the propagation of waves, including compressive shock-waves and self-similar expansion waves. Yet since the 1930's, the modern theory of cosmology has been based on the starting assumption of the Copernican Principle, which restricts the whole theory to the Friedmann spacetimes, a special class of solutions of the Einstein equations which describe a uniform three space of constant curvature and constant density evolving in time. Our approach has been to look for general relativistic waves that could perturb a uniform Friedmann background.

The family of GR self-similar expanding waves derived in [**17**] satisfy two important conditions: they include and perturb the Standard Model of Cosmology, and they take the form of non-interacting time asymptotic wave patterns during the radiation phase of the Big Bang when a mechanism is in place for the decay of complicated solutions to simple wave forms. That is, quantitative theories of Lax and Glimm describe how solutions of conservation laws decay in time to self-similar wave patterns, explaining how *entropy*, *shock-wave dissipation* and *time-irreversibility*, (concepts originally understood only in the context of ideal gases), could be given meaning in general systems of *conservation laws*, a setting much more general than gas dynamics. (This viewpoint is well expressed in the celebrated works [**12, 7, 8**]). The conclusion: Shock-waves introduce dissipation and increase of entropy into the dynamics of solutions, and this provides a mechanism by which complicated solutions can settle down to orderly self-similar wave patterns, *even when dissipative terms are neglected in the formulation of the equations*. A rock thrown into a pond demonstrates how the mechanism can transform a chaotic "plunk" into a series of orderly outgoing self-similar waves moments later–but the self-similar waves here are more like the uniform expansion set in motion by a tsunami. As a result, our new construction of a family of GR self-similar waves that apply when this decay mechanism should be in place, received a good deal of media attention when it came out in PNAS, August 2009.

The expanding wave solutions in our family are determined by a single parameter a, the *acceleration parameter*, which is normalized so that at $a = 1$ the solution reduces exactly to the critical $k = 0$ FRW spacetime of the Standard Model with pure radiation sources. When $a \neq 1$, we show that solutions look remarkably similar to $k = 0$ FRW, and prove that the spacetimes in the family are distinct from all the other non-critical FRW spacetimes $k \neq 0$, so the critical FRW spacetime during the radiation phase is *characterized* as the unique spacetime lying at the intersection of these two one parameter families. More importantly, adjustment of the free parameter a speeds up or slows down the expansion rate relative to the Standard Model according to whether $a > 1$ or $a < 1$, respectively. Based on this, we argue that these self-similar waves can account for the leading order quadratic correction to redshift vs luminosity observed in the super nova data, without the need for Dark Energy.

Temple first proposed the idea that the anomalous acceleration might be accounted for by a wave in the talk *Numerical Shock-wave Cosmology*, New Orleans, January 2007, and he set out with students to simulate such a wave numerically. While attempting to set up the numerical simulation of such a wave, we discovered that the Standard Model during the radiation phase admits a coordinate system (Standard Schwarzschild Coordinates (SSC)) in which the Friedmann spacetime is *self-similar*. That is, it took the form of a non-interacting time-asymptotic wave pattern according to the theory of Lax and Glimm. This was the key. Once we found this, we guessed that the Einstein equations in these coordinates must close to form a new system of ODE's in the same self-similar variable. After deriving this system of equations, we verified that the standard model was represented as one point in the family of solutions of these equations parameterized by the three initial conditions. Symmetry and regularity at the center then reduced the three parameter family to an implicitly defined one parameter family, one value of which ($a = 1$) gives the critical Friedmann spacetime of the Standard Model during the radiation phase of the Big Bang.

Our idea then: an expansion wave that formed during the radiation epoch, when the Einstein equations obey a highly nonlinear system of conservation laws for which we must expect self-similar non-interacting waves to be the end state of local fluctuations, could account for the anomalous acceleration of the galaxies without Dark Energy. Since we have explicit formulas for such waves, it is a verifiable proposition.

In [**17**] we derived the equations for the expanding waves and recorded the quadratic correction to redshift vs luminosity which they imply for an observer fixed at the center of the wave. In the longer paper [**20**], which is to appear in Memoirs of the AMS, we supplied details, and derived the third order correction term. The third order correction term must take account of the dimming of light from a distant galaxy due to the curvature of spacetime in the self-similar waves when $a \neq 1$. This is a subtle effect not present in the Standard Model, and not effecting the calculation of the quadratic correction term. In fact the calculation of the third order term was greatly simplified by Etherington's Theorem, which was brought to our attention by the referee of the paper. To start, we summarize the results in [**17**] in the following theorems. (Unbarred coordinates (t, r) refer to FRW co-moving coordinates, and barred coordinates $(\bar{t}, \bar{r})$ refer to (SSC).)

Theorem 1. *Assume $p = \frac{1}{3}\rho c^2$, $k = 0$ and $R(t) = \sqrt{t}$. Then the FRW metric*

$$ds^2 = -dt^2 + R(t)^2 dr^2 + \bar{r}^2 d\Omega^2, \tag{1.1}$$

under the change of coordinates

$$\bar{t} = \psi_0 \left\{ 1 + \left[\frac{R(t)r}{2t} \right]^2 \right\} t, \tag{1.2}$$

$$\bar{r} = R(t)r, \tag{1.3}$$

transforms to the SSC-metric

$$ds^2 = -\frac{d\bar{t}^2}{\psi_0^2 \left(1 - v^2(\xi)\right)} + \frac{d\bar{r}^2}{1 - v^2(\xi)} + \bar{r}^2 d\Omega^2, \tag{1.4}$$

where

$$v = \frac{1}{\sqrt{AB}} \frac{\bar{u}^1}{\bar{u}^0} \tag{1.5}$$

is the SSC velocity, which also satisfies

$$v = \frac{\zeta}{2}, \tag{1.6}$$

$$\psi_0 \xi = \frac{2v}{1+v^2}. \tag{1.7}$$

THEOREM 2. *Let ξ denote the self-similarity variable*

$$\xi = \frac{\bar{r}}{\bar{t}}, \tag{1.8}$$

and let

$$G = \frac{\xi}{\sqrt{AB}}. \tag{1.9}$$

Assume that $A(\xi)$, $G(\xi)$ and $v(\xi)$ solve the ODE's

$$\xi A_\xi = -\left[\frac{4(1-A)v}{(3+v^2)G-4v}\right] \tag{1.10}$$

$$\xi G_\xi = -G\left\{\left(\frac{1-A}{A}\right)\frac{2(1+v^2)G-4v}{(3+v^2)G-4v} - 1\right\} \tag{1.11}$$

$$\xi v_\xi = -\left(\frac{1-v^2}{2\{\cdot\}_D}\right)\left\{(3+v^2)G-4v+\frac{4\left(\frac{1-A}{A}\right)\{\cdot\}_N}{(3+v^2)G-4v}\right\}, \tag{1.12}$$

where

$$\{\cdot\}_N = \left\{-2v^2+2(3-v^2)vG-(3-v^4)G^2\right\} \tag{1.13}$$

$$\{\cdot\}_D = \left\{(3v^2-1)-4vG+(3-v^2)G^2\right\}, \tag{1.14}$$

and define the density by

$$\kappa\rho = \frac{3(1-v^2)(1-A)G}{(3+v^2)G-4v}\frac{1}{\bar{r}^2}. \tag{1.15}$$

Then the metric

$$ds^2 = -B(\xi)d\bar{t}^2 + \frac{1}{A(\xi)}d\bar{r}^2 + \bar{r}^2 d\Omega^2 \tag{1.16}$$

solves the Einstein-Euler equations $G = \kappa T$ with velocity $v = v(\xi)$ and equation of state $p = \frac{1}{3}\rho c^2$. In particular, the FRW metric (1.4) solves equations (1.10)-(1.12).

The main point is that the coordinate mapping (1.2)-(1.3) taking co-moving FRW coordinates over to SSC coordinates, explicitly identifies the self-similar variables as well as the metric ansatz that together accomplish a self-similar extension of the FRW metric. In particular, it is not evident from (1.1) alone that self-similar variables even exist, and if they do exist, by what ansatz one should extend the metric in those variables to obtain nearby self-similar solutions that solve the Einstein equations exactly.

Removing time-scaling invariance and imposing regularity at the center, we prove in [**17, 20**] that the family of solutions of (1.10)-(1.12) parameterized by three initial conditions, reduces to an implicitly defined one parameter family of self-similar expanding waves determined by a parameter a normalized so that $a = 1$ is the FRW Standard Model. Thus, adjustment of parameter a away from $a = 1$ changes the expansion rate of the spacetimes in the family relative to the Standard Model. The following theorem gives exact formulas for this family of spacetime metrics in FRW coordinates, up to fourth order in the variable ζ which measures fractional distance from the center of the wave to the Hubble length, (the distance of light travel since the Big Bang)–again, the fractional distance ζ is measured back in the comoving coordinates of the original FRW spacetime of the Standard Model, c.f. [**20**]:

Theorem 3. *The inverse of the coordinate transformation (1.2)-(1.3) maps the one parameter family of self-similar spacetimes defined implicitly by equations (1.10)-(1.12) over to the (t, r)-coordinate metric*

$$ds^2 = F_a(\zeta)^2 \left\{-dt^2 + tdr^2\right\} + tr^2 d\Omega^2, \tag{1.17}$$

where $\zeta = \bar{r}/t$, $\bar{r} = \sqrt{t}r =$ distance as measured at fixed t in the Standard Model,

$$F_a(\zeta)^2 = 1 + (a^2 - 1)\frac{\zeta^2}{4} + O(1)|a - 1|\zeta^4, \tag{1.18}$$

and the SSC velocity v maps to the (t, r)-velocity

$$w = -\frac{a^2 - 1}{8}\zeta^3 + O(1)|a - 1|\zeta^4. \tag{1.19}$$

Again, ζ is distance at fixed time divided by time since the Big Bang in the Standard Model, so ζ measures fractional distance to the Hubble length in FRW comoving coordinates (t, r).

The resulting one parameter family of metrics is amenable to the calculation of a redshift vs luminosity relation. The quadratic correction to redshift factor z, first established in [**17**], is recorded in the following theorem which applies during the radiation phase of the expansion:

Theorem 4. *The redshift vs luminosity relation, as measured by an observer positioned at the center of the expanding wave spacetimes (metrics of form (1.16)), is given up to third order in redshift factor z by*

$$d_\ell = 2ct\left\{z + \frac{a^2 - 1}{2}z^2 + O(1)|a - 1|z^3\right\}, \tag{1.20}$$

where d_ℓ is luminosity distance, ct is invariant time since the Big Bang, and a is the acceleration parameter that distinguishes expanding waves in the family.

Note first that (1.20) reduces to the correct linear relation of the Standard Model when $a = 1$, c.f., [**10**]. When $a \neq 1$, (1.20) shows that different solutions in the family expand at different rates according to the value of the parameter a, and adjustment of this parameter speeds up or slows down the expansion rate during the radiation phase, thereby altering the redshift vs luminosity distance relation relative to the FRW Standard Model at the quadratic level.

The specific redshift vs luminosity distance relations recorded in (1.20), while correct for a radiation dominated universe, will not be the precise relations valid for an observer in the later universe after it has cooled to a point where non-relativistic matter dominates its energy density. But continuity of the subsequent evolution with respect to parameters implies that the leading order correction associated with an arbitrary (small) anomalous acceleration observed after the radiation phase of Standard Model, could be accounted for by adjustment of the parameter a. Indeed, because the equation of state $p = \frac{c^2}{3}\rho$ is the equation of state for both pure radiation and matter in the extreme relativistic limit, the displacement of the co-moving frames from the Standard Model in an expanding wave during the radiation phase, would induce a corresponding displacement in the co-moving frames of the matter field at the end of the radiation epoch, and this displacement would evolve in time as the pressure drops. Thus, to conclude that (sufficiently small) leading order corrections to redshift vs luminosity could be accounted for after the radiation phase by adjustment of the parameter a, all that is required is that the quadratic correction to the evolving redshift vs luminosity relation have a continuous and monotonic dependence on a near $a = 1$. Making this precise is the topic of the authors current research.

2. Etherington's Theorem and the Third Order Term

The third order correction to redshift vs luminosity is important because it is at third order that the expanding wave theory can be tested against experiment. A calculation based on Etherington's theorem will appear in the authors' forthcoming paper [**20**]. We now record this improvement to (1.20) in the following theorem:

THEOREM 5. *The third order correction to (1.20) is given by*

$$d_\ell = 2ct\left\{z + \frac{a^2-1}{2}z^2 + \frac{(a^2-1)(a^2+2)}{2}z^3 + O(1)|a-1|z^4\right\}. \tag{2.1}$$

The main problem in determining the third order term in (5) is to estimate the ratio C_a of an area $\mathcal{A}$ of light received from a distance source at a mirror (telescope) positioned at the origin when $a \neq 1$, to the corresponding area when $a = 1$, in the limit $\mathcal{A} \to 0$, the limit expressing that the mirror is small relative to the distance to the source. We call this the *mirror problem.* The factor C_a reflects spacetime curvature in the expanding waves, and $C_a = 1$ in the Standard Model $a = 1$. Since the expanding spacetimes when $a \neq 1$ agree with the Standard Model near the center up to second order in fractional distance to the Hubble length, it follows that the effect of C_a enters the calculation of redshift vs luminosity only at third order in redshift factor z, and hence it does not affect the second order correction (5) first recorded in [**17**]. We now summarize the resolution of the mirror problem by Etherington's theorem, as will appear in [**20**].

To describe the mirror problem, consider light emitted from a distant source located at (t_e, r_e) and received at a mirror of area $\mathcal{A}$ positioned orthogonal to the line of sight at the center $r = \zeta = 0$ of our spherically symmetric expanding spacetimes (1.17), at a later time $t = t_0 > t_e > 0$. The problem is to determine the fraction $f_{\mathcal{A}}$ of the area of the 2-sphere emitting radiation at $r = r_e$, $t = t_e$ that reaches the mirror. In the case of the Standard Model $a = 1$, the center of the FRW (t, r)-coordinate system can be translated to any point. Taking the center to be (t_e, r_e), light rays leaving the source at (t_e, r_e) will follow radial geodesics $d\Omega = 0$.

LEMMA 1. *For the Stardard Model $a = 1$, the area of the unit 2-sphere emitting radiation at $r = r_e$, $t = t_e$ that reaches the mirror $\mathcal{A}$ is $f_{\mathcal{A}} = \mathcal{A}/4\pi t_0 r_e^2$.*

Proof: To see this, consider a packet of lightlike radial geodesics covering angular area $d\Omega$, emanating from an FRW coordinate center at $r = r_e$, and evolving up to an end at time $t = t_0$. Such curves, being radial lightlike geodesics, traverse the curves $\hat{r} = 2\left(\sqrt{t} - \sqrt{t_e}\right)$, $\theta = \theta_0 \in d\Omega$, $t_e \leq t \leq t_0$, where $\hat{r}$ is radial distance measured from the new center r_e, and θ measures angles at center r_e. Now setting $\xi = \sqrt{t} - \sqrt{t_0}$, these curves project into the curves at time $t \equiv t_0$ given by $\hat{r} = 2\,(\xi)$, $\theta = \theta_0$, $0 \leq \xi \leq r_e/2$. Since in the Standard Model, $t = t_0$ is flat Euclidean space, the latter curves, being at fixed time $t = t_0$, are just the straight lines in $\mathcal{R}^3$ emanating from center $\hat{r} = 0$, sweeping out the angular region $d\Omega$ at $\hat{r} = 0$ and the area $\mathcal{A}$ at $\hat{r} = r_e$, $r = 0$. It thus follows that the area at the end is $\mathcal{A} = \bar{r}_e^2 d\Omega$, where $\bar{r_e} = R(t_0) r_e = \sqrt{t_0} r_e$ is spatial distance at received time $t = t_0$. So the fractional area is $f_{\mathcal{A}} = d\Omega/4\pi = \mathcal{A}/4\pi t_0 r_e^2$, as claimed. $\Box$

When $a \neq 1$, the 3-spaces at fixed time are not *homogeneous* and *isotropic* about every point like the $a = 1$ FRW, and the geodesics leaving the center of a coordinate system centered at (t_e, r_e) will not follow $d\Omega = 0$ exactly. So there is a correction factor C_a required in the formula for $f_{\mathcal{A}}$ when $a \neq 1$; namely,

$$f_{\mathcal{A}} = C_a \mathcal{A}/4\pi t_0 r_e^2.$$

The determination of C_a is made simpler by Etherington's Theorem [**6**], (also referred to as the Reciprocity Theory, c.f. [**13**], pages 256-259), which we state as follows:

THEOREM 6. (Etherington, 1933): *Assume that light emitted from a galaxy at spacetime point G is received at spacetime point O with redshift z observed at O. Then*

$$\frac{\delta S_O}{d\Omega_G} = \frac{\delta S_G}{d\Omega_O}(1+z)^2, \tag{2.2}$$

where δS_O is the (infinitessimal) area of a mirror positioned orthogonal to the received light rays at O, $d\Omega_G$ is the angular area of the bundle of light rays emitted at G that reach the mirror δS_O, and δS_G is a reciprocal area, positioned at G orthogonal to the light rays from G to O, with $d\Omega_O$ the corresponding angular area of backward time light rays emitted at O, whose backward time trajectories intersect the area δS_G.

The theorem applies to any gravitational spacetime metric subject only to the condition that the bundle of light rays received at δS_O completely surround O, such that there are rays of the bundle in every direction from O within δS_O, c.f. Figure 16.2, page 256 of [**13**]. The result is motivated by the observation that the backward time light rays from O are affected at the same spacetime points by the same spacetime metric as the forward time light rays from G, so there must be a relation between the corresponding areas and angles, and that relation is quantified by (2.2).

To find C_a, let the light ray from G to O be the radial null geodesic taking $G = (t_e, r_e)$ to $O = (t_0, 0)$ for the spacetime metric (1.17) depending on parameter a. Then Etherington's theorem gives

$$\frac{\delta S_O^a}{d\Omega_G^a} = \frac{\delta S_G^a}{d\Omega_O^a}(1+z_a)^2, \tag{2.3}$$

where z_a is the redshift observed at O, depending on a through the metric (1.17). Now since C_a is the ratio of an area δS_O^a of light received at O when $a \neq 1$, to the corresponding area δS_O^1 received at O when $a = 1$, it follows that

$$C_a = \frac{\delta S_O^a}{\delta S_O^1}. \tag{2.4}$$

Dividing (2.3) at $a \neq 1$ by (2.3) at $a = 1$ then gives the formula

$$C_a = \frac{\delta S_G^a}{\delta S_G^1} \frac{(1 + z_a)^2}{(1 + z_1)^2}.$$

Since angles are constant along radial geodesics of (1.17) emanating from the center O, for every a, in both forward and backward time it follows that

$$\frac{\delta S_G^a}{\delta S_G^1} = 1,$$

so

$$C_a = \frac{(1 + z_a)^2}{(1 + z_1)^2}.$$

The mirror problem and the determination of C_a is the main effect of spacetime curvature on the third order correction term in (2.1). See [**20**] for further details. □

3. Our Program for Future Research

Our program now is to obtain the quadratic and cubic corrections to redshift vs luminosity induced by the expanding waves at present time, by evolving forward, up through the $p = 0$ stage of the Standard Model, the corrections (2.1) for the expanding wave perturbations at the end of the radiation phase. Matching the leading order correction to the data will fix the choice of acceleration parameter, and the third order correction, at that choice of acceleration parameter, is then a verifiable prediction of the theory. This, again, is a topic of the authors' current research.

These results suggest an interpretation that we might call a *Conservation Law* explanation of the anomalous acceleration of the galaxies. That is, the theory of Lax and Glimm explains how highly interactive oscillatory solutions of conservation laws will decay in time to non-interacting waves, (shock waves and expansion waves), by the mechanisms of wave interaction and shock wave dissipation. The subtle point is that even though dissipation terms are neglected in the formulation of the equations, there is a canonical dissipation and consequent loss of information due to the *nonlinearities*, and this can be modeled by shock wave interactions that drive solutions to non-interacting wave patterns. Since the one fact most certain about the Standard Model is that our universe arose from an earlier hot dense epoch in which all sources of energy were in the form of radiation, and since it is approximately uniform on the largest scale but highly oscillatory on smaller scales[2], one might reasonably conjecture that decay to a non-interacting expanding wave occurred during the radiation phase of the Standard Model, via the highly nonlinear evolution driven by the large sound speed, and correspondingly large modulus of *genuine nonlinearity*[3], present when $p = \rho c^2/3$, c.f. [**14**]. Our analysis has shown

[2]In the Standard Model, the universe is approximated by uniform density on a scale of a billion light years or so, about a tenth of the radius of the visible universe, [**22**]. The stars, galaxies and clusters of galaxies are then evidence of large oscillations on smaller scales.

[3]Again, *genuine nonlinearity* is in the sense of Lax, a measure of the magnitude of nonlinear compression that drives decay, c.f., [**12**].

that FRW is just one point in a family of non-interacting, self-similar expansion waves, and as a result we conclude that some further explanation is required as to why, on some length scale, decay during the radiation phase of the Standard Model would not proceed to a member of the family satisfying $a \neq 1$. If decay to $a \neq 1$ did occur, then the galaxies that formed from matter at the end of the radiation phase, (some $379,000$ years after the Big Bang), would be displaced from their anticipated positions in the Standard Model at present time, and this displacement would lead to a modification of the observed redshift vs luminosity relation. In short, the displacement of the fluid particles, (i.e., the displacement of the co-moving frames in the radiation field), by the wave during the radiation epoch leads to a displacement of the galaxies at a later time. In principle such a mechanism could account for the anomalous acceleration of the galaxies as observed in the supernova data.

If $a \neq 1$, then the spacetime within the expansion wave has a center, and this would violate the so-called *Copernican Principle*, a simplifying assumption generally accepted in cosmology, at least on the scale of the wave (c.f. the discussions in [**21**] and [**1**]). Moreover, if our Milky Way galaxy did not lie within some threshold of the center of expansion, the expanding wave theory would imply unobserved angular variations in the expansion rate. In fact, all of these observational issues have already been discussed recently in [**2, 1, 3**], (and references therein), which explore the possibility that the anomalous acceleration of the galaxies might be due to a local *void* or under-density of galaxies in the vicinity of the Milky Way.[4] Our proposal then, is that the one parameter family of general relativistic self-similar expansion waves derived here are possible end-states that could result after dissipation by wave interactions during the radiation phase of the Standard Model is completed, and such waves could thereby account for the appearance of a local under-density of galaxies at a later time.

In any case, the expanding wave theory is testable. For a first test, we propose next to evolve the quadratic and cubic corrections to redshift vs luminosity recorded here in relation (1.20), valid at the end of the radiation phase, up through the $p \approx 0$ stage to present time in the Standard Model, to obtain the present time values of the quadratic and cubic corrections to redshift vs luminisity implied by the expanding waves, as a function of the acceleration parameter a. Once accomplished, we can look for a best fit value of a via comparison of the quadratic correction at present time to the quadratic correction observed in the supernova data, leaving the third order correction at present time as a prediction of the theory. That is, in principle, the predicted third order correction term could be used to distinguish the expanding wave theory from other theories (such as dark energy) by the degree to which they match an accurate plot of redshift vs luminosity from the supernove data, (a topic of the authors' current research). The idea that the anomalous acceleration might be accounted for by a local under-density in a neighborhood of our galaxy was expounded in the recent papers [**2, 3**]. Our results here might then give an accounting for the source of such an under-density.

The expanding wave theory could in principle give an explanation for the observed anomalous acceleration of the galaxies within classical general relativity,

[4]The size of the center, consistent with the angular dependence that has been observed in the actual supernova and microwave data, has been estimated to be about 15 megaparsecs, approximately the distance between clusters of galaxies, roughly $1/200$ the distance across the visible universe, c.f. [**1, 2, 3**].

with classical sources. In the expanding wave theory, the so-called anomalous acceleration is not an acceleration at all, but is a correction to the Standard Model due to the fact that we are looking outward into an expansion wave. The one parameter family of non-interacting, self-similar, general relativistic expansion waves derived here, are all possible end-states that could result by wave interaction and dissipation due to nonlinearities back when the universe was filled with pure radiation sources. And when $a \neq 1$ they introduce an anomalous acceleration into the Standard Model of cosmology. Unlike the theory of Dark Energy, this provides a possible explanation for the anomalous acceleration of the galaxies that is not *ad hoc* in the sense that it is derivable exactly from physical principles and a mathematically rigorous theory of general relativistic expansion waves. In particular, this explanation does not require the *ad hoc* assumption of a universe filled with an as yet unobserved form of energy with anti-gravitational properties, (the standard physical interpretation of the cosmological constant), in order to fit the data.

In summary, these new general relativistic expanding waves provide a new paradigm to test against the Standard Model. Even if they do not in the end explain the anomalous acceleration of the galaxies, one has to believe they are present and propagating on some scale, and their presence represents an instability in the Standard Model in the sense that an explanation is required as to why small scale oscillations have to settle down to large scale $a = 1$ expansions instead of $a \neq 1$ expansions, (either locally or globally), during the radiation phase of the Big Bang.

References

[1] C. Copi, D. Huterer, D.J. Schwarz & G.D. Starkman, *On the large-angle anomalies of the microwave sky*, Mon. Not. R. Astron. Soc., (2005) pp. 1-26.

[2] T. Clifton, P.G. Ferreira & K. Land, *Living in a void: testing the Copernican principle with distant supernovae*, Phys. Rev. Lett., **101**, (2008), 131302 (arXiv:0807.1443v2 [astro-ph] 29 Sep 2008).

[3] T. Clifton & P.G. Ferreira, *Does dark energy really exist?*, Sci. Am., **2009**, April (2009), 48–55.

[4] E. Coddington & N. Levinson, *Theory of Ordinary Differential Equations*, (page 330, Theorem 4), McGraw Hill, (1955).

[5] D. Eardley, *Self-similar spacetimes: Geometry and dynamics*, Commun. Math. Phys, **37**, 287-309 (1974).

[6] I.M.H. Etherington, *Republication of: LX. On the definition of distance in general relativity*, Gen Relativ Gravit, **39**, 1055-1067 (2007).

[7] J. Glimm, *Solutions in the large for nonlinear hyperbolic systems of equations*, Comm. Pure Appl. Math., **18**, 697-715 (1965).

[8] J. Glimm & P.D. Lax, *Decay of solutions of systems of nonlinear hyperbolic conservation laws*, Memoirs Amer. Math Soc. **101** (1970).

[9] J. Groah & B. Temple, *Shock-wave solutions of the Einstein equations: Existence and consistency by a locally inertial Glimm Scheme*, Memoirs of the AMS, Vol. 172, No. 813, November 2004.

[10] O. Gron & S. Hervik, *Einstein's General Theory of Relativity with Modern Applications in Cosmology*, Springer Verlag, 2007.

[11] Hartman, *A lemma in the structural stability of ordinary differential equations*, Proc. Amer. Math. Soc, 11, 610-620 (1960).

[12] P.D. Lax, *Hyperbolic systems of conservation laws, II*, Comm. Pure Appl. Math., **10**, 537–566 (1957).

[13] J. Plebanski & A. Krasinski, *An Introduction to General Relativity and Cosmology*, Cambridge University Press, (2006).

[14] J. Smoller & B. Temple, *Global solutions of the relativistic Euler equations*, Comm. Math. Phys., **157**, 67-99 (1993).

[15] J. Smoller & B. Temple, *Cosmology, black holes, and shock waves beyond the Hubble length*, Meth. Appl. Anal., **11**, 77-132 (2004).

[16] J. Smoller & B. Temple, *Shock-wave cosmology inside a black hole*, Proc. Nat. Acad. Sci., September 30, 2003. Vo. 100, no. 20, (11216–11218).

[17] B. Temple & J. Smoller, *Expanding wave solutions of the Einstein equations that induce an anomalous acceleration into the Standard Model of Cosmology*, Proc. Nat. Acad. Sci., August 25, 2009, Vol. 106, no. 34, (14213–14218).

[18] B. Temple & J. Smoller, *Answers to Questions Posed by Reporters: Temple-Smoller GR expanding waves*, August 19, 2009. http://www.math.ucdavis.edu/temple/.

[19] J. Smoller & B. Temple, General relativistic self-similar waves that induce an anomalous acceleration into the Standard Model of Cosmology, arXiv:submit/0044499 [gr-qc] 20 May 2010.

[20] J. Smoller & B. Temple, General relativistic self-similar waves that induce an anomalous acceleration into the Standard Model of Cosmology, Memoirs of the AMS, (to appear).

[21] B. Temple, *Numerical Refinement of a Finite Mass Shock-Wave Cosmology*, AMS National Meeting, Special Session *Numerical Relativity*, New Orleans (2007): http://www.math.ucdavis.edu/∼temple/talks/NumericalShockWaveCosTalk.pdf.

[22] S. Weinberg, *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity*, John Wiley & Sons, New York, 1972.

Department of Mathematics, University of Michigan, Ann Arbor, MI 48109

Department of Mathematics, University of California, Davis, Davis CA 95616