

**COURSE DESCRIPTION:** This is the beginning upper division class on differential geometry. *Differential Geometry* is the modern extension of Euclid's *Plane Geometry*. The program Euclid put forth in the third century BC was to develop all of the truths of geometry from a set of five axioms, but work of Gauss, Lobachevski and Bolya in the early nineteenth century showed that these axioms were too limited to describe the curved spaces that are all around us, including the space and time we actually inhabit. Modern differential geometry is based on the starting assumption that lengths and angles are determined by a *metric* that encodes how to compute them, and within this framework one can define *curvature*. Gauss defined the curvature of a two dimensional surface and introduced the notion of a metric in 1831, but it was Riemann in 1855 who gave the general (formidable!) definition of curvature in spaces of dimension 3 and higher. The greatest achievement of all came in 1915, when Albert Einstein derived the equations of general relativity, the modern theory of gravitation. In Einstein's theory, the gravitational field is not a *Force* (Newton), but is actually the manifestation of *curvature* as measured by the *Riemann curvature tensor*. But to make his theory work, the curvature had to be in space *and* time, which together he called *spacetime*. Riemann's theory of curvature and Einstein's theory of general relativity are deep subjects beyond the level of this class. But in this class we will introduce the foundations of differential geometry, starting with the geometry of vectors and curves, and then surfaces, all with many applications. Two of the goals will be to understand the *metric* on a surface, and in terms of this the Gaussian curvature, foreshadowings of the great theories of Riemann and Einstein.