HOMEWORK SOLUTIONS

(2.6.2) The point is that we can integrate $\int_t^{t+T} f(x) \frac{dx}{dt} dt$ two different ways to get a contradiction. The first way, recall $f(x) = -U'(x)$ where the antiderivative $U(x)$ is called the potential. Then since $(dx/dt)dt = dx$ we can use the substitution principle to get

$$\int_t^{t+T} f(x) \frac{dx}{dt} dt = \int_t^{t+T} f(x) dx = \int_{x(t)}^{x(t+T)} (-U'(x)) dx$$

$$= -[U(x(t + T)) - U(x(t))] = 0,$$  \hspace{1cm} (1)

because by assumption $x(t + T) = x(t)$.

On the other hand, $dx/dt = f(x)$, so

$$\int_t^{t+T} f(x) \frac{dx}{dt} dt = \int_t^{t+T} [f(x(t))]^2 dt > 0,$$  \hspace{1cm} (2)

because the integral of a positive function is positive. The two statements cannot both be true, so the assumption that the solution is periodic, (i.e., $x(t) = x(t + T)$), must be false. (Proof by contradiction!)