Introduction/ Preview of Sect. 5.1

Review - II

Lecture 3

1

We can view AB as (I) The colms of B acting on A 2 ways or (II) The rows of A acting on B 2 ways

(I)

① Colms of B acting on rows of A:

$$AB = \text{"}(r_i \cdot c_j)\text{"} = AB$$

② Colms of B acting on colms of A:

Ex

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$= \left[\begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right]$$

$$= \left[\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}(1) + \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}(-1) + \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}(0), \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}(0) + \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}(1) + \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}(1) \right]$$

"The first colm of AB is the linear combination of the colms of A determined by the 1st colm of B"

" The 2nd ...

... 2nd colm of B "

In general:

$$\begin{bmatrix} | & & | \\ a_1 & \cdots & a_k \\ | & & | \end{bmatrix} \cdot \begin{bmatrix} | & & | \\ b_1 & \cdots & b_n \\ | & & | \end{bmatrix} = \begin{bmatrix} | & & | \\ c_1 & \cdots & c_n \\ | & & | \end{bmatrix}$$

$$\begin{bmatrix} | \\ a_1 \\ | \end{bmatrix} x_1 + \begin{bmatrix} | \\ a_2 \\ | \end{bmatrix} x_2 + \cdots + \begin{bmatrix} | \\ a_n \\ | \end{bmatrix} x_n = \begin{bmatrix} | \\ c_j \\ | \end{bmatrix}$$

$$b_j = (x_1, \cdots, x_n)$$

(II) View $A \cdot B$ as the rows of A acting on ① colums of B or ② rows of B

① $A \cdot B = \underset{A}{\begin{bmatrix} - & r_1 & - \\ & \vdots & \\ - & r_m & - \end{bmatrix}} \underset{B}{\begin{bmatrix} | & & | \\ c_1 & \cdots & c_n \\ | & & | \end{bmatrix}} = \underset{AB}{\text{``}(r_i c_j)\text{''}}$

"i row of A dots j colum of B to given (i,j)th entry of AB."

2 View AB as the rows of A acting on rows of B . . .

Ex

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$= \left[\begin{array}{c} 1\cdot[1,0] + 2[-1,1] + (-1)[0,1] \\ \hline 0\cdot[1,0] + (-1)\cdot[-1,1] + (1)\cdot[0,1] \\ \hline (1)[1,0] + (1)[-1,1] + (1)[0,1] \end{array}\right] \begin{array}{l} \} \text{ 1st row} \\ \} \text{ 2nd row} \\ \} \text{ 3rd row} \end{array}$$

In general:

$$\begin{bmatrix} - a_1 - \\ - a_2 - \\ \vdots \\ - a_m - \end{bmatrix} \begin{bmatrix} - b_1 - \\ - b_2 - \\ \vdots \\ - b_k - \end{bmatrix} = \begin{bmatrix} - c_1 - \\ - c_2 - \\ \vdots \\ - c_m - \end{bmatrix}$$

$$A_{m \times k} \qquad B_{k \times n} \qquad C_{m \times n}$$

$$\begin{bmatrix} c_i \end{bmatrix} = \begin{bmatrix} x_1 [- b_1 -] \\ + \\ x_2 [- b_2 -] \\ + \\ \vdots \\ + \\ x_k [- b_k -] \end{bmatrix}$$

$$\uparrow$$

$$a_i = (x_1, \ldots, x_k)$$

Note: $(AB)C = A(BC)$

but **Not** $AB = BA$

except in special cases

"The i'th row of A determines the linear combination of the rows of B that gives the ith row of AB"