

MIDTERM EXAM I
Math 16A
Temple-Winter 2010

–Print your name, section number and put your signature on the upper right-hand corner of this exam. Write only on the exam.

–Show all of your work, and justify your answers for full credit.

SCORES

#1

#2

#3

#4

#5

#6

#7

#8

TOTAL:

1. Determine the following limits:

(a) (6 pts) $\lim_{x \rightarrow 2} \sqrt{\frac{x^2-4}{x-2}}$ (Hint: Factor)

(b) (6 pts) $\lim_{x \rightarrow 0} \frac{\sqrt{x+2}-\sqrt{2}}{4x}$ (Hint: Conjugate)

2. Determine the following limits:

(c) (6 pts) $\lim_{x \rightarrow +\infty} \frac{2x^5 + 3x - 4}{3x^5 - 7}$ (Hint: Divide by highest power)

(d) (6 pts) $\lim_{x \rightarrow 0} \frac{1 - \cos^2 x}{\tan^2 x \cos x}$ (Hint: Simplify)

3. Let $f(x) = \sqrt{4 - x^2}$, $g(x) = 2 \sin x$.

(a) (5 pts) Find the Domain of f .

(b) (5 pts) Find the (precise) Range of g .

(c) (6 pts) Find $(f \circ g)(x)$.

(d) (5 pts) Prove that $(f \circ g)(x)$ is defined for every real number x . (I.e, show the Domain of $f \circ g$ is all of \mathcal{R} .)

4. Consider the function $f(x) = \frac{2x+1}{x-3}$ with Domain $x \neq 3$.

(a) (7 pts) Find a formula for $f^{-1}(x)$.

(b) (4 pts) Find the Domain of f^{-1} .

(c) (4 pts) Evaluate $f^{-1}(1)$

5. Find all vertical and horizontal asymptotes (you needn't graph the functions):

(a) (3 pts) $y = \sin x$

(b) (6 pts) $y = \frac{2x^2}{(x^2-1)(x+2)}$

(c) (6 pts) $y = \tan x$

6. Given a function $f(x)$:

(a) (4 pts) State the *definition* of the derivative $f'(x)$.

(b) (7 pts) Directly from the definition, derive the value of $f'(2)$ if $f(x) = x^3$. (Hint: $(x - a)^3 = x^3 - 3ax^2 + 3a^2x - a^3$).

7. (7 pts) Draw the graph of a function continuous except at $x = 2$, such that $f(2) = 1$, $\lim_{x \rightarrow 2^-} f(x) = 0$, and $\lim_{x \rightarrow 2^+} f(x) = 4$. (Use closed and open dots correctly at $x = 2$.)

8. (7 pts) Find the equation of the line *perpendicular* to the graph of $y = 2x^2 - 3$ at the point $(1, -1)$.