MIDTERM EXAM I
Math 16A
Temple-Winter 2012

- Print your name, section number and put your signature on the upper right-hand corner of this exam. Write only on the exam.

- Show all of your work, and justify your answers for full credit.

SCORES

#1

#2

#3

#4

#5

#6

#7

TOTAL:
1. Let \( f(x) = \left( \frac{x^2-16}{x-4} \right)^{12} \). Evaluate the following limits: (Do not simplify)

(a) (6 pts) \( \lim_{x \to 2} \left( \frac{x^2-16}{x-4} \right)^{12} = \left( \frac{2^2 - 16}{2 - 4} \right)^{12} \)

(b) (6 pts) \( \lim_{x \to 4} \left[ \frac{x^2-16}{x-4} \right]^{12} = \left[ \frac{(x-4)(x+4)}{x-4} \right]^{12} = \left( \frac{8}{1} \right)^{12} \)
2. (7 pts) Find all vertical and horizontal asymptotes and sketch the graph of the function \( f(x) = \frac{3x+1}{x-1} \). Justify your answer.

**Vertical** \[ x = 1 \]

**Horizontal** \[ \lim_{x \to \infty} \frac{3x+1}{x-1} = 3 \quad \lim_{x \to -\infty} \frac{3x+1}{x-1} = -3 \]

\[ y = 3 \]
3. (7 pts) Find all vertical asymptotes and sketch the graph of the function \( f(x) = \tan(x) \) for \( 0 \leq x \leq 2\pi \).

\[
f'(x) = \tan x = \frac{\sin x}{\cos x} \quad \cos x = 0 \iff x = \frac{\pi}{2}, \frac{3\pi}{2}
\]

Vert asymptotes \( x = \frac{\pi}{2}, \frac{3\pi}{2} \)
4. (7 pts) Let \( f(x) = 2x^2 \). Find the equation of the line passing through the two points on its graph \( (1, f(1)) \) and \( (2, f(2)) \). Sketch a graph the function \( f \) and the line.

\[
m = \frac{8 - 2}{2 - 1} = 6
\]

\[
\text{Eqn:} \quad \frac{y - 2}{x - 1} = 6 \quad \Rightarrow \quad y - 2 = 6x - 6
\]

\[
y = 6x - 4
\]
5. (7 pts) Use the definition of derivative to find the slope \( \frac{dy}{dx} = f'(2) \) of the line tangent to the graph of \( f(x) = 2x^2 \) at the point \( (2, f(2)) \). Sketch the graph and the tangent line.

\[
\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}
\]

\[
= \lim_{\Delta x \to 0} \left[ \frac{2(2 + \Delta x)^2 - 2(2)^2}{\Delta x} \right]
\]

\[
= \lim_{\Delta x \to 0} \left[ \frac{2(4 + 4\Delta x + \Delta x^2) - 8}{\Delta x} \right]
\]

\[
= \lim_{\Delta x \to 0} \left[ \frac{8 + 8\Delta x + 2\Delta x^2}{\Delta x} \right] = 8
\]

[No credit for \( f'(x) = 4x \)]
6. (a) (7 pts) Give a formula for a function $f(x)$ that is not continuous at $x = 2$, but such that $\lim_{x \to 2} f(x) = 3$.

$$f(x) = 3 \frac{x-2}{x-2}$$

(b) (4 pts) Give a formula for a function $f(x)$ such that $f$ is continuous at $x \neq 2$, but $\lim_{x \to 2} f(x)$ does not exist.

$$f(x) = \frac{1}{x-2}$$
7. Let $f(x) = \frac{1}{4-x}$.

(a) (5 pts) What is the Domain of $f$.

\[ x \neq 4 \]

(b) (5 pts) Find a formula for the inverse $f^{-1}$.

\[
y = \frac{1}{4-x} \Rightarrow x = \frac{1}{4-y} \Rightarrow 4-y = \frac{1}{x}
\]

\[ y = 4 - \frac{1}{x} = f^{-1}(x) \]

(c) (6 pts) Find the Domain of $(f \circ f)(x) = f(f(x))$.

\[
f(f(x)) = \frac{1}{4 - \left(\frac{1}{4-x}\right)} = \frac{4-x}{4(4-x)-1}
\]

\[
= \frac{4-x}{16-4x-1} = \frac{4-x}{15-4x}
\]

Domain \[ 15-4x \neq 0 \]

\[ x \neq \frac{15}{4} \]