MIDTERM EXAM II
Math 16A
Temple-Fall 2012

−Print your name and put your signature on the upper right-hand corner of this exam. Write only on the exam.

−Show all of your work, and justify your answers for full credit.

SCORES

#1

#2

#3

#4

#5

#6

#7

TOTAL:

1. Differentiate: (Do not simplify.)

(a) (4 pts) $y = \frac{1270}{730} - 2x^{21} + 4x^{11}$

$f'(x) =$

(b) (4 pts) $f(x) = \{\tan(x)\}\{\sin x\}$

$f'(x) =$

(c) (4 pts) $y = \frac{2x^3-3}{3x^2+1}$

$f'(x) =$

(d) (4 pts) f(x)=sin (x^4+1)

$f'(x) =$

2. (15 pts) Differentiate: $f(x) = \frac{\sin^3(x+\sqrt{x})}{x \tan x}$ (Do not simplify.)

$f'(x) =$

3. Assume the height y in feet of a falling object after t seconds is given by

$$y = -16t^2 + 32t + 48.$$

(a) (4 pts) Find the velocity $v = \frac{dy}{dt}$ as a function of t.

(b) (4 pts) Find the acceleration $a = \frac{dv}{dt}$ as a function of t.

(c) (4 pts) Find the velocity at $t = 0$.

(d) (4 pts) Find the highest the object goes. (Hint: $v = \frac{dy}{dt} = 0$ at the moment when the highest point is reached.)

4. The cost of making x jet airplanes in millions of dollars is

$$C(x) = 4x - \sqrt{x}.$$

Recall that the marginal cost of producing dx more airplanes than x is dy where

$$\frac{dy}{dx} = C'(x).$$

(a) (6 pts) Find the cost of producing 100 airplanes.

(b) (8 pts) Find the marginal cost of producing two more airplanes if $x = 100$. (Hint: Solve for dy.)

5. (13 pts) Using the definition of derivative,

$$\frac{d}{dx}f(x) \equiv f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x},$$

give a careful proof that $\frac{d}{dx}x^2 = 2x$.

6. (13 pts) Using the definition of derivative,

$$\frac{d}{dx}f(x) \equiv f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x},$$

verify the product rule:

$$\frac{d}{dx}\{f(x)g(x)\} = f'(x)g(x) + f(x)g'(x).$$

7. (13 pts) Using the definition of derivative,

$$\frac{d}{dx}f(x) \equiv f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x},$$

give a careful proof that $\frac{d}{dx}\sin(x) = \cos(x)$. You may use the fact that

$$\lim_{\Delta x \to 0} \frac{\sin(\Delta x)}{\Delta x} = 1,$$

and

$$\lim_{\Delta x \to 0} \frac{1 - \cos(\Delta x)}{\Delta x} = 0.$$