MIDTERM EXAM II–SOLUTIONS
Math 16A
Temple-Winter 2010

−Print your name, section number and put your signature on the upper right-hand corner of this exam. Write only on the exam.

−Show all of your work, and justify your answers for full credit.

<u>SCORES</u>

#1

#2

#3

#4

#5

#6

#7

TOTAL:

1. Differentiate: (Do not simplify.)

   (a) (4 pts) $y = -x^{13} + 6x^2 - \frac{127}{73}$ (Hint: Power Rule)

   $y' = -13x^{12} + 12x$

   (b) (4 pts) $f(x) = \{\sin(x)\}\{\cos x\}$ (Hint: Product Rule)

   ****Note:** $f(x) = \{\sin(x)\}\{\cos(x)\}$ was corrected on board during exam****************

   $y' = (\cos x)(\cos x) + (\sin x)(-\sin x) = \cos^2 x - \sin^2 x$

   (c) (4 pts) $y = \frac{2x^2-3}{x^3+1}$ (Hint: Quotient Rule)

   $y' = \frac{(x^3+1)(4x)-(2x^2-3)(3x^2)}{(x^3+1)^2}$

   (d) (4 pts) f(x)=$\sin(2x^2+1)$ (Hint: Chain Rule)

   $y' = \cos(2x^2+1)\,4x$

2. (10 pts) Differentiate: $f(x) = \frac{\tan^2(x+\sqrt{x})}{x\sin x}$ (Do not simplify.)

$$f'(x) = \frac{(x\sin x)(2\tan(x+\sqrt{x})\ \sec^2(x+\sqrt{x})\,(1+\frac{1}{2}x^{-1/2}) - (\tan^2(x+\sqrt{x}))(\sin x + x\cos x)}{(x\sin x)^2}$$

3. (10 pts) A population of bacteria is introduced into a culture. The number of bacteria P can be modeled by

$$P = 450\left(1 + \frac{t}{5+t^2}\right),$$

where t is in hours. Find the rate of population growth after one hour.

Solultion: Rate=$\frac{dP}{dt} = 450\left(\frac{(5+t^2)-2t^2}{(5+t^2)^2}\right)$. At $t = 1$ the rate is:

$$P'(1) = 450 \cdot \frac{6-2}{36} = 450\frac{1}{9} = 50 \;\; \textit{per hour.}$$

4. Let $y = f(x)$ where $f(x) = \frac{1}{3}x^3 - x^2 - 3x + 1$.

(a) (3 pts) Find $f'(x)$.

Solution: $f'(x) = x^2 - 2x - 3 = (x-3)(x+1)$.

(b) (3 pts) Use brackets to describe where f is *increasing*

Solution: $f'(x) > 0$ when $x \in (-\infty, -1) \cup (3, \infty)$.

(c) (3 pts) Use brackets to describe where f is *decreasing*

Solution: $f'(x) < 0$ when $x \in (-1, 3)$.

(d) (3 pts) Find (x, y) where f takes a (local) *maximum*.

Solution: $x = -1, \quad y = -\frac{1}{3} - 1 + 3 + 1 = 2\frac{2}{3}$.

(e) (3 pts) Find (x, y) where f takes a (local) *minimum*.

Solution: $x = 3, \quad y = 9 - 9 - 9 + 1 = -8$.

(f) (3 pts) Sketch the graph of f labeling (b)-(e).

Solution: "Up from $-\infty$ to $2\frac{2}{3}$ at $x = -1$, back down to -8 at $x = 3$, back up to ∞."

5. (15 pts) A ball is thrown upward and forward. Its vertical height (in feet) after t seconds is given by

$$y(t) = -16t^2 + 8t.$$

Note: Misprint on exam $y(t) = 16t^2 - 8t$ was corrected on board during exam*****************

Before hitting the ground it moves horizontally forward at constant velocity $\frac{dx}{dt} = 4$ feet per second.

(a) (5 pts) Find a formula for the vertical velocity $\frac{dy}{dt}$.

Solution: $v = \frac{dy}{dt} = -32t + 8$.

(b) (5 pts) Find a formula for the vertical acceleration.

Solution: $a = -32\ ft/sec^2$

(c) (5 pts) Find the highest the ball goes up.

Solution: $-32t + 8 = 0$ so $t = 8/32 = 1/4$ sec., so $y_{max} = -16\left(\frac{1}{4^2}\right) + 8\frac{1}{4} = -1 + 2 = 1$ft.

(d) (5 pts) Find the time at which the ball hits the ground.

Solution: $y = -16t^2 + 8t = 8t(-2t + 1) = 0$ so $t = 0, 1/2$.

(e) (5 pts) Find how far the ball traveled horizontally forward when it hit the ground.

Solution: Horizontal velocity =4 feet per second, it travels for 1/2 second, so $d = (1/2)(4) = 2$ ft.

6. Consider the following complicated equation:

$$x^2y^3 - 2xy - 1 = 0. \tag{1}$$

(a) (8 pts) Obtain a formula for $\frac{dy}{dx}$ the slope of the line tangent to the graph of (1) at a point (x, y) on the graph.

Solution: (Implicit Differentiation)

$$\begin{aligned} 2xy^3 + 3x^2y^2\frac{dy}{dx} - 2y - 2x\frac{dy}{dx} &= 0 \\ \left\{3x^2y^2 - 2x\right\}\frac{dy}{dx} + 2xy^3 - 2y = 0. \end{aligned}$$

Solving for $\frac{dy}{dx}$ gives:

$$\frac{dy}{dx} = \frac{-2xy^3 + 2y}{3x^2y^2 - 2x}$$

(b) (8 pts) Find the equation of the line tangent to the graph of (1) at the point $(1, -1)$.

Solution: At $(x, y) = (1, -1)$,

$$\frac{dy}{dx} = \frac{2(1)(-1)^3 - 2(-1)}{2 - 3(1)^2(-1)^2} = 0.$$

So the equation of the line is:

$$\frac{y - (-1)}{x - 1} = 0,$$

or

$$y = -1.$$

7. (15 pts) A car is moving parallel and along the shore of a straight river three miles wide. A policeman on the shore across the river from the car uses radar to determine that at the moment the car is 5 miles from him, it is receding from him at a velocity of 120 miles an hour. Draw a correct picture, label the relevant variables, and determine the velocity of the car at the moment it is 5 miles away from the policeman. (Hint: draw a picture and let $x(t)$ denote the diagonal distance from the policeman to the car, and let $y(t)$ denote the distance from the car to the point directly across the river from the policeman.)

   ****Note:** The clarification *The rate at which the distance between the car and the policeman was increasing was 120mph* was written on the board during exam*****************

   Solution: $y(t)^2 + 3^2 = x(t)^2$ so $2yy' = 2xx'$, and so

$$y' = \frac{xx'}{y} = \frac{(5)(120)}{4} = 150 \; mph$$