

MIDTERM EXAM II
Math 16A
Temple-Winter 2010

–Print your name, section number and put your signature on the upper right-hand corner of this exam. Write only on the exam.

–Show all of your work, and justify your answers for full credit.

SCORES

#1

#2

#3

#4

#5

#6

#7

TOTAL:

1. Differentiate: (Do not simplify.)

(a) (4 pts) $y = -x^{13} + 6x^2 - \frac{127}{73}$ (Hint: Power Rule)

$$y' =$$

(b) (4 pts) $f(x) = \{\sin(x)\} \{\cos x\}$ (Hint: Product Rule)

$$y' =$$

(c) (4 pts) $y = \frac{2x^2-3}{x^3+1}$ (Hint: Quotient Rule)

$$y' =$$

(d) (4 pts) $f(x) = \sin(2x^2 + 1)$ (Hint: Chain Rule)

$$y' =$$

2. (10 pts) Differentiate: $f(x) = \frac{\tan^2(x+\sqrt{x})}{x \sin x}$ (Do not simplify.)

$$f'(x) =$$

3. (10 pts) A population of bacteria is introduced into a culture. The number of bacteria P can be modeled by

$$P = 450 \left(1 + \frac{t}{5 + t^2} \right),$$

where t is in hours. Find the rate of population growth after one hour.

4. Let $y = f(x)$ where $f(x) = \frac{1}{3}x^3 - x^2 - 3x + 1$.

(a) (3 pts) Find $f'(x)$.

(b) (3 pts) Use brackets to describe where f is *increasing*

(c) (3 pts) Use brackets to describe where f is *decreasing*

(d) (3 pts) Find (x, y) where f takes a (local) *maximum*.

(e) (3 pts) Find (x, y) where f takes a (local) *minimum*.

(f) (3 pts) Sketch the graph of f labeling (b)-(e).

5. A ball is thrown upward and forward. Its vertical height (in feet) after t seconds is given by

$$y(t) = 16t^2 - 8t.$$

Before hitting the ground it moves horizontally forward at constant velocity $\frac{dx}{dt} = 4$ feet per second.

- (a) (3 pts) Find a formula for the vertical velocity $\frac{dy}{dt}$.
- (b) (3 pts) Find a formula for the vertical acceleration.
- (c) (3 pts) Find the highest the ball goes up.
- (d) (3 pts) Find the time at which the ball hits the ground.
- (e) (3 pts) Find how far the ball traveled horizontally forward when it hit the ground.

6. Consider the following complicated equation:

$$x^2y^3 - 2xy - 1 = 0. \quad (1)$$

(a) (8 pts) Obtain a formula for $\frac{dy}{dx}$ the slope of the line tangent to the graph of (1) at a point (x, y) on the graph.

(b) (8 pts) Find the equation of the line tangent to the graph of (1) at the point $(1, -1)$.

7. (15 pts) A car is moving parallel and along the shore of a straight river three miles wide. A policeman on the shore across the river from the car uses radar to determine that at the moment the car is 5 miles from him, it is receding from him at a velocity of 120 miles an hour. Draw a correct picture, label the relevant variables, and determine the velocity of the car at the moment it is 5 miles away from the policeman. (Hint: draw a picture and let $x(t)$ denote the diagonal distance from the policeman to the car, and let $y(t)$ denote the distance from the car to the point directly across the river from the policeman.)