

1.) (12 pts.) Use a Three-Step Process to determine if the following function is continuous at $x = 1$.

$$f(x) = \begin{cases} x^2 + 2x, & \text{if } x \leq 1 \\ 3 - x^2, & \text{if } x > 1 \end{cases}$$

i.) $f(1) = (1)^2 + 2(1) = 3$

ii.) $\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (x^2 + 2x) = 1 + 2 = 3$

and $\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} (3 - x^2) = 3 - 1 = 2$

so $\lim_{x \rightarrow 1} f(x)$ DNE
and f is NOT continuous at $x = 1$.

2.) Consider the function $f(x) = \frac{x^2}{x^2 - 9}$

a.) (4 pts.) Use limits to determine all horizontal asymptotes (H.A.).

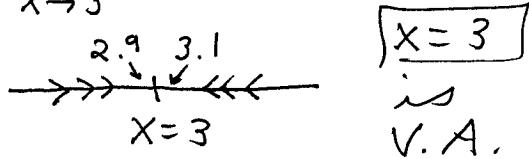
$$\lim_{x \rightarrow \pm\infty} \frac{x^2}{x^2 - 9} \cdot \frac{1/x^2}{1/x^2} = \lim_{x \rightarrow \pm\infty} \frac{1}{1 - 9/x^2} = \frac{1}{1 - 0} = 1$$

so $\boxed{Y=1}$ is H.A.

b.) (8 pts.) Use limits to determine all vertical asymptotes (V.A.).

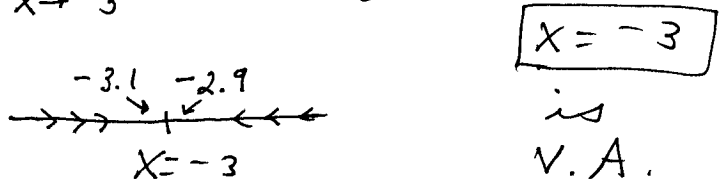
$$\lim_{x \rightarrow 3^+} f(x) = \frac{9}{0^+} = +\infty$$

$$\lim_{x \rightarrow 3^-} f(x) = \frac{9}{0^-} = -\infty$$



$$\lim_{x \rightarrow -3^+} f(x) = \frac{9}{0^-} = -\infty$$

$$\lim_{x \rightarrow -3^-} f(x) = \frac{9}{0^+} = +\infty$$



c.) (6 pts.) Use x - and y -intercepts and the above information to sketch a graph of f .

$$x=0: y=0$$

$$y=0: \frac{x^2}{x^2-9} = 0$$

$$\rightarrow x^2 = 0 \rightarrow x=0$$

