Name: ______________________________

Student ID#: ________________________

Section: ________________________

Midterm Exam 1

Friday, February 1

MAT 185A, Temple, Winter 2019

Print names and ID's clearly, and have your student ID ready to be checked when you turn in your exam. Write the solutions clearly and legibly. Do not write near the edge of the paper or the stapled corner. Show your work on every problem. Correct answers with no supporting work will not receive full credit. Be organized and use notation appropriately. No calculators, notes, books, cellphones, etc. may be used on this exam.

Problem	Your Score	Maximum Score
1		20
2		20
3		20
4		20
5		20
Total		100

Problem #1 (20pts): Recall that $e^{i\theta} = \cos\theta + i\sin\theta$. Prove that

$$e^{i(\theta_1+\theta_2)} = e^{i\theta_1}e^{i\theta_2}$$

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Problem #2 (20pts): Let $\vec{G} = \overrightarrow{(M, N)}$ be a vector field in the plane, where $M = M(x, y)$, $N = N(x, y)$ are real valued functions of (x, y). Let C be a smooth curve in the plane taking point A to point B. Use Leibniz's substitution principle to show the following are equal: (Here $\mathbf{r}(t) = \overrightarrow{(x(t), y(t))}$ denotes any smooth parameterization of curve C.)

$$\int_C \overrightarrow{G} \cdot \overrightarrow{T}\ ds = \int_C M dx + N dy = \int_C \overrightarrow{G} \cdot d\overrightarrow{\mathbf{r}} = \int_C \overrightarrow{G} \cdot \overrightarrow{\mathbf{v}}\ dt.$$

Problem #3 (20pts): Prove that $f(z) = 1/z$ is analytic (its complex derivative exists) for all $z = x + iy \neq 0$ two ways:

(1) By showing directly $\lim_{\Delta z \to 0} \frac{f(z+\Delta z) - f(z)}{\Delta z}$ exists.

(2) By proving $1/z = u(x, y) + iv(x, y)$ satisfies the Cauchy-Riemann equation $u_x = v_y$, $u_y = -v_x$.

Problem #4 (20pts): Assume $f(z) = u(x, y) + iv(x, y)$ be complex differentiable for all $z \in \mathcal{C}$, (so the Cauchy-Riemann equations hold), and let C be a curve that takes A to B.

(a) Derive, in terms of u and v, formulas for the real valued vector fields $\overrightarrow{G_1} = \overrightarrow{(M_1, N_1)}$ and $\overrightarrow{G_2} = \overrightarrow{(M_2, N_2)}$ such that

$$\int_C f(z)dz = \int_C \overrightarrow{G_1} \cdot \overrightarrow{T} ds + i \int_C \overrightarrow{G_2} \cdot \overrightarrow{T} ds,$$

where $\int_C \overrightarrow{G_i} \cdot \overrightarrow{T} ds$ are the corresponding real line integrals on $\mathcal{R}^2$.

(b) Use the Cauchy-Riemann equations to prove that $\overrightarrow{G_1}$ and $\overrightarrow{G_2}$ are curl free, and state a theorem which implies that there exist $U(x, y)$ and $V(x, y)$ such that

$$\overrightarrow{G_1} = \nabla U, \quad \overrightarrow{G_2} = \nabla V.$$

(c) Letting $F(z) = U + iV$, prove

$$\int_C f(z)dz = F(B) - F(A).$$

(You may use any theorem from Mat21D which you can state correctly.)

(d) Prove $F(z)$ satisfies the Cauchy-Riemann equations, and $F'(z) = f(z)$.

Problem #5 (20pts): Let C denote the simple closed curve given by the unit circle centered at $z = 0$, oriented counterclockwise around $z = 0$. Evaluate $\int_C \frac{dz}{z}$ directly by parameterization.