Name: ________________________________

Student ID#: __________________________

Section: __________________________

Midterm Exam 2

Wednesday, March 4

MAT 185A, Temple, Winter 2020

Print names and ID's clearly, and have your student ID ready to be checked when you turn in your exam. Write the solutions clearly and legibly. Do not write near the edge of the paper or the stapled corner. Show your work on every problem. Correct answers with no supporting work will not receive full credit. Be organized and use notation appropriately. No calculators, notes, books, cellphones, etc. may be used on this exam.

Problem	Your Score	Maximum Score
1		20
2		20
3		20
4		20
5		20
Total		100

Problem #1 (20pts): Recall that $\cos z = \frac{e^{iz}+e^{-iz}}{2}$, for $z = x + iy \in \mathcal{C}$.

(a) Show that $\cos z$ reduces to $\cos x$ when $z = x \in \mathcal{R}$.

(b) Find $u(x, y)$ and $v(x, y)$ real so that $\cos(z) = u(x, y) + iv(x, y)$.

(c) Prove $f(z) = \cos z$ satisfies the Cauchy-Riemann equations $u_x = v_y$, $u_y = -v_x$.

Problem #2 (20pts): Find all of the values of $(i)^{\frac{1}{\pi}}$. How many are there? Justify.

Problem #3 (20pts): Let $\mathcal{C}_{\mathcal{R}}$ denote the positively oriented circle of radius R center z, and let $\overline{B_R(z)}$ denote the closed ball of radius R and center z. Recall the Cauchy Integral Formula tells us that if f is analytic in a neighborhood of $\overline{B_R(z)}$, then

$$f(z) = \frac{1}{2\pi i} \int_{\mathcal{C}_{\mathcal{R}}} \frac{f(w)}{w - z} dw.$$

(a) Prove that if f is analytic in a neighborhood of $\overline{B_R(z)}$, then $|f'(z)| \leq \frac{M}{R}$, where M is the maximum value of f in $\overline{B_R(z)}$.

(b) Prove Liouville's Theorem, that every bounded entire function is constant.

Problem #4 (20pts): Assume that $f(z) = u + iv$ is analytic everywhere.

(a) Prove that $\Delta u = 0$ and $\Delta v = 0$ where $\Delta u = u_{xx} + v_{yy}$.

(b) Prove $u(0) = \frac{1}{2\pi} \int_0^{2\pi} u(e^{it}) dt$. (Hint: Integrate Cauchy Integral Formula on the unit circle.)

Problem #5 (20pts): Let $P(z) = a_n z^n + \cdots + a_1 z + a_0$ be a complex polynomial, where $a_n \neq 0$ and $n \geq 1$

(a) Prove that there exists $R > 0$ such that $P(z) \geq 2$ for $|z| > R$.

(b) Prove that $P(z)$ has at least one root z_0 such that $P(z_0) = 0$.