Take Home Final Exam
MAT 185A, Temple, Winter 2022
Due Friday, March 18, 2022, at 5 pm.

(A) The assignment is to present a self contained discussion of the Maximum Principle for Harmonic Functions in the plane.

(1) Start with a correct statement (without proof) of Cauchy Integral Formula.

(2) Assuming Part (1), prove that any analytic function at z is the "average" of its values on any circle of radius r center z. Use Riemann sums to explain why it is the average.

(3) Derive the corresponding "averaging principle" for the real and imaginary parts u and v of an analytic function. Prove that u, v are *harmonic*, i.e., satisfy $\Delta u = 0 = \Delta v$.

(4) Use (1)-(3) to give a careful proof that the real part of a harmonic function can never take a strict maximum at an interior point (x_0, y_0). I.e., we cannot have $|u(x, y)| < u(x_0, y_0)$ for all (x, y) in a deleted neighborhood of (x_0, y_0) where f is analytic.

(5) Give a careful proof that every harmonic function u has a harmonic conjugate v such that $f(z) = u + iv$ is analytic.

(B) The assignment is to present a self contained discussion of the Residue Theorem as follows:

(1) Start with a correct statement (without proof) of the Lorentz Expansion theorem for analytic functions in an annulus $r < |z| < R$, stating conditions under which the series converges uniformly, and describing when one can pass integrals through summation signs.

(2) Assuming Part (1), derive the Residue Theorem for meromorphic functions from the Cauchy Integral Formula.

(3) Present the five applications of the Residue Theorem to evaluating real integrals which I presented in Lecture Notes VIII, IX, X posted on Professor's webpage.

Please make this the cover page for your Final Exam, and upload your exam in PDF to *Gradescope*, (NOT to TA or Professor!), by 5 pm, Freday, March 18, 2022.

Please enjoy understanding this and writing this up in your own words, as the Maximum Principle and Residue Theorem of Complex Variables are two of the greatest discoveries in the history of mathematics.

But Note: This is NOT a group project, and to get credit the writeup must be **in your own words**. Collaborating on a Final Exam is considered cheating in the University of California. You are however free to email specific questions to TA or Professor.