(3) b) IF $\cos z = 4$, THEN $\dfrac{e^{iz}+e^{-iz}}{2} = 4$ SO $e^{iz}+e^{-iz} = 8$.

THEN $e^{iz} - 8 + e^{-iz} = 0$, SO $e^{2iz} - 8e^{iz} + 1 = 0$. (MULT. BY e^{iz})

THEN $e^{iz} = \dfrac{8 \pm \sqrt{60}}{2} = 4 \pm \sqrt{15}$ BY THE QUADRATIC FORMULA,

SO $iz = \text{LOG}\,(4 \pm \sqrt{15})$.

1) IF $iz = \text{LOG}\,(4+\sqrt{15}) = \text{LN}\,(4+\sqrt{15}) + 2n\pi i$, THEN $\boxed{z = 2n\pi - \text{LN}\,(4+\sqrt{15})\,i}$ (MULT. BY $-i$)

2) IF $iz = \text{LOG}\,(4-\sqrt{15}) = \text{LN}\,(4-\sqrt{15}) + 2n\pi i$, THEN $\boxed{z = 2n\pi - \text{LN}\,(4-\sqrt{15})\,i}$ (MULT. BY $-i$)

SINCE $(4+\sqrt{15})(4-\sqrt{15}) = 1$, $\text{LN}\,(4+\sqrt{15}) + \text{LN}\,(4-\sqrt{15}) = 0$ SO $\text{LN}\,(4-\sqrt{15}) = -\text{LN}\,(4+\sqrt{15})$

AND WE CAN ALSO WRITE THE ANSWER AS $\boxed{z = 2n\pi \pm \text{LN}\,(4+\sqrt{15})\,i}$, $n \in \mathbb{Z}$

OR IF $\cos z = 4$, THEN $\sin^2 z = 1 - \cos^2 z = 1 - 16 = -15$ SO $\sin z = \sqrt{-15} = \pm\sqrt{15}\,i$

THEN $e^{iz} = \cos z + i \sin z = 4 \mp \sqrt{15}$, SO $iz = \text{LOG}\,(4 \pm \sqrt{15})$

NOW PROCEED AS ABOVE.

(7) a) $(-i)^i = e^{i\,\text{LOG}(-i)} = e^{i\left[\text{LN}\,1 + \left(-\frac{\pi}{2} + 2n\pi\right)i\right]} = e^{-\left(-\frac{\pi}{2}+2n\pi\right)} = \boxed{e^{\frac{\pi}{2} - 2n\pi}} = \boxed{e^{\pi/2}\, e^{-2n\pi}}$, $n \in \mathbb{Z}$

REMARK WE COULD USE $\frac{3\pi}{2}$ IN PLACE OF $-\frac{\pi}{2}$, OR ANY OTHER COTERMINAL ANGLE.

b) $(1+i)^{1+i} = e^{(1+i)\,\text{LOG}\,(1+i)} = e^{(1+i)\left[\text{LN}\sqrt{2} + \left(\frac{\pi}{4} + 2n\pi\right)i\right]}$

$= e^{\left(\text{LN}\sqrt{2} - \frac{\pi}{4} - 2n\pi\right)}\, e^{i\left(\text{LN}\sqrt{2} + \frac{\pi}{4} + 2n\pi\right)}$

$= \boxed{\sqrt{2}\, e^{-\frac{\pi}{4} - 2n\pi}\left[\cos\left(\text{LN}\sqrt{2} + \frac{\pi}{4}\right) + i \sin\left(\text{LN}\sqrt{2} + \frac{\pi}{4}\right)\right]}$, $n \in \mathbb{Z}$ (USING $e^{i\theta} = e^{i(\theta + 2n\pi)}$)

(9) IF $z = a + bi$, THEN

$\overline{e^{iz}} = \overline{e^{-b+ai}} = \overline{e^{-b}(\cos a + i \sin a)} = e^{-b}(\cos a - i \sin a)$ AND

$e^{i\bar{z}} = e^{i(a-bi)} = e^{b+ai} = e^{b}(\cos a + i \sin a)$, SO

$\overline{e^{iz}} = e^{i\bar{z}}$ IFF $e^{-b} = e^{b}$ AND $\sin a = -\sin a$ IFF $-b = b$ AND $\sin a = 0$

IFF $b = 0$ AND $a = n\pi$ IFF $\boxed{z = n\pi}$, $n \in \mathbb{Z}$.

(16) a) $\cosh^2 z - \sinh^2 z = \left(\dfrac{e^{z}+e^{-z}}{2}\right)^2 - \left(\dfrac{e^{z}-e^{-z}}{2}\right)^2$

$= \dfrac{e^{2z} + 2 + e^{-2z}}{4} - \dfrac{e^{2z} - 2 + e^{-2z}}{4} = \dfrac{4}{4} = 1.$

(17) IF $z = x + yi$, $\sin z = \sin(x + yi) = \sin x \cos(yi) + \sin(yi)\cos x$

$= \sin x \cosh y + i \sinh y \cos x$

SO $|\sin z|^2 = \sin^2 x \cosh^2 y + \sinh^2 y \cos^2 x$

$= \sin^2 x \cosh^2 y + (\cosh^2 y - 1)\cos^2 x$

$= \cosh^2 y\,(\sin^2 x + \cos^2 x) - \cos^2 x = \cosh^2 y - \cos^2 x \le \cosh^2 y$,

AND THEREFORE $|\sin z| \le |\cosh y|$.

(SIMILARLY, $|\sinh y| \le |\sin z|$.)

(18) If $b \in \mathbb{R}$, then $a^b = e^{b \log a} = e^{b[\ln|a| + i \arg a]} = e^{b \ln|a|} \cdot e^{i(b \arg a)}$ where $b \arg a \in \mathbb{R}$,

so $|a^b| = e^{b \ln|a|} = (e^{\ln|a|})^b = |a|^b$.

(24) a) Image of $x = a$: if $z = a + yi$, then

$$\cos z = \cos(a + yi) = \cos a \cos yi - \sin a \sin yi$$
$$= \cos a \cosh y - \sin a\,(i \sinh y) = u + iv$$

where $u = \cos a \cosh y$ and $v = -\sin a \sinh y$.

Since $\dfrac{u^2}{\cos^2 a} - \dfrac{v^2}{\sin^2 a} = \cosh^2 y - \sinh^2 y = 1$,

The image of a vertical line is a $\boxed{\text{HYPERBOLA}}$. (Right or left branch)

$\boxed{\text{OR}}$ use $\cos z = \dfrac{e^{iz} + e^{-iz}}{2} = \dfrac{e^{-y+ai} + e^{y-ai}}{2} = \frac{1}{2}\left(e^{-y}(\cos a + i \sin a) + e^{y}(\cos a - i \sin a)\right)$

$= u + iv$ where $u = \dfrac{\cos a}{2}(e^{-y} + e^{y})$ and $v = \dfrac{\sin a}{2}(e^{-y} - e^{y})$,

so $\dfrac{u^2}{\cos^2 a} - \dfrac{v^2}{\sin^2 a} = \left(\dfrac{e^{-y} + e^{y}}{2}\right)^2 - \left(\dfrac{e^{-y} - e^{y}}{2}\right)^2 = 1$

b) Image of $y = b$: if $z = x + bi$, then

$$\cos z = \cos(x + bi) = \cos x \cos bi - \sin x \sin bi$$
$$= \cos x \cosh b - \sin x\,(i \sinh b) = u + vi$$

where $u = \cos x \cosh b$ and $v = -\sin x \sinh b$.

Since $\dfrac{u^2}{\cosh^2 b} + \dfrac{v^2}{\sinh^2 b} = \cos^2 x + \sin^2 x = 1$,

The image of a horizontal line is an $\boxed{\text{ELLIPSE}}$.

$\boxed{\text{OR}}$ use $\cos z = \dfrac{e^{iz} + e^{-iz}}{2} = \dfrac{e^{-b+xi} + e^{b-xi}}{2} = \frac{1}{2}\left(e^{-b}(\cos x + i \sin x) + e^{b}(\cos x - i \sin x)\right)$

$= u + iv$ where $u = \dfrac{\cos x}{2}(e^{-b} + e^{b})$ and $v = \dfrac{\sin x}{2}(e^{-b} - e^{b})$,

so $\dfrac{u^2}{\left(\frac{e^{-b} + e^{b}}{2}\right)^2} + \dfrac{v^2}{\left(\frac{e^{-b} - e^{b}}{2}\right)^2} = \cos^2 x + \sin^2 x = 1$.

REMARKS A) 1) The image of $x = a$ is the right or left branch of the hyperbola, depending on whether $\cos a > 0$ or $\cos a < 0$.

2) The image of $x = a$ is the imaginary axis if $\cos a = 0$, and it is the interval $[1, \infty)$ or $(-\infty, -1]$ on the real axis if $\sin a = 0$ (depending on whether $\cos a = 1$ or $\cos a = -1$).

B) The image of $y = b$ is the interval $[-1, 1]$ on the real axis if $b = 0$.