# Section 8.1

1.) $\int_0^1 \frac{16x}{8x^2+2}\,dx = \ln|8x^2+2|\Big|_0^1$

$= \ln 10 - \ln 2 = \ln\frac{10}{2} = \ln 5$

2.) $\int \frac{x^2}{x^2+1}\,dx = \int \frac{x^2+1-1}{x^2+1}\,dx$

$= \int\left(1 - \frac{1}{x^2+1}\right)dx = x - \arctan x + C$

3.) $\int (\sec x - \tan x)^2\,dx$

$= \int (\sec^2 x - 2\sec x\tan x + \tan^2 x)\,dx$

$= \int (\sec^2 x - 2\sec x\tan x + (\sec^2 x - 1))\,dx$

$= \tan x - 2\sec x + \tan x - x + C$

4.) $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1}{\cos^2 x\tan x}\,dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1}{\cos^{\cancel{2}} x\cdot\frac{\sin x}{\cancel{\cos x}}}\,dx$

$= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{2}{2\sin x\cos x}\,dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{2}{\sin 2x}\,dx$

$= 2\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \csc(2x)\,dx = \ln|\csc(2x) - \cot(2x)|\Big|_{\frac{\pi}{4}}^{\frac{\pi}{3}}$

$= \ln|\csc(\tfrac{2}{3}\pi) - \cot(\tfrac{2}{3}\pi)| - \ln|\csc(\tfrac{\pi}{2}) - \cot(\tfrac{\pi}{2})|$

$= \ln\left|\frac{2}{\sqrt{3}} - \frac{-1}{\sqrt{3}}\right| - \ln|\cancel{1}^{0} - 0| = \ln\sqrt{3}$

5.) $\int \frac{1-x}{\sqrt{1-x^2}}\,dx = \int\left[\frac{1}{\sqrt{1-x^2}} - \frac{x}{(1-x^2)^{1/2}}\right]dx$

$= \arcsin x - \cancel{2}\cdot\frac{-1}{\cancel{2}}(1-x^2)^{1/2} + C$

$= \arcsin + \sqrt{1-x^2} + c$

6.) $\int \frac{1}{x-\sqrt{x}}\,dx = \int \frac{1}{\sqrt{x}(\sqrt{x}-1)}\,dx$ (Let $u=\sqrt{x}-1$

$\rightarrow du = \frac{1}{2}x^{-1/2}\,dx$

$\rightarrow 2\,du = \frac{1}{\sqrt{x}}\,dx$)

$= 2\int \frac{1}{u}\,du$

$= 2\ln|u| + c$

$= 2\ln|\sqrt{x}-1| + c$

9.) $\int \frac{1}{e^z+e^{-z}}\,dz = \int \frac{1}{e^z+e^{-z}} \cdot \frac{e^z}{e^z}\,dz$

$= \int \frac{e^z}{(e^z)^2+1}\,dz = \arctan(e^z) + c$

10.) $\int_1^2 \frac{8}{x^2-2x+2}\,dx = \int_1^2 \frac{8}{(x-1)^2+1}\,dx$

$= 8\arctan(x-1)\Big|_1^2$

$= 8\arctan 1 - 8\arctan 0$

$= 8\cdot\left(\frac{\pi}{4}\right) - 8\cdot(0) = 2\pi$

13.) $\int \frac{1}{1-\sec t}\,dt = \int \frac{1+\sec t}{(1-\sec t)(1+\sec t)}\,dt$

$= \int \frac{1+\sec t}{1-\sec^2 t}\,dt = \int \frac{1+\sec t}{-\tan^2 t}\,dt$

$= -\int \left( \cot^2 t + \frac{\frac{1}{\cos t}}{\frac{\sin^2 t}{\cos^2 t}} \right) dt$

$$= -\int \left[ (\csc^2 t - 1) + \frac{\cos t}{(\sin t)^2} \right] dt$$

$$= -\left( -\cot t - t - \frac{1}{\sin t} \right) dt$$

14.) $\int \csc t \cdot \sin 3t \, dt = \int \csc t \cdot \sin(t + 2t) \, dt$

$$= \int \frac{1}{\sin t} (\sin t \cdot \cos 2t + \cos t \cdot \sin 2t) \, dt$$

$$= \int \left( \cos 2t + \frac{1}{\cancel{\sin t}} \cdot \cos t \cdot 2 \cancel{\sin t} \cos t \right) dt$$

$$= \frac{1}{2} \sin 2t + \int 2\cos^2 t \, dt$$

$$= \frac{1}{2} \sin 2t + \int (\cos 2t + 1) \, dt$$

$$= \frac{1}{2} \sin 2t + \frac{1}{2} \sin 2t + t + C$$

$$= \sin 2t + t + C$$

16.) $\int \frac{1}{\sqrt{2\theta - \theta^2}} \, d\theta = \int \frac{1}{\sqrt{-(\theta^2 - 2\theta)}} \, d\theta$

$$= \int \frac{1}{\sqrt{1 - (\theta^2 - 2\theta + 1)}} \, d\theta = \int \frac{1}{\sqrt{1 - (\theta - 1)^2}} \, d\theta$$

$$= \arcsin(\theta - 1) + C$$

18.) $\int \frac{2^{\sqrt{y}}}{2\sqrt{y}} \, dy$ (Let $u = \sqrt{y} \rightarrow du = \frac{1}{2} y^{-1/2} dy$

$du = \frac{1}{2\sqrt{y}} \, dy$)

$$= \int 2^u \, du$$

$$= \frac{1}{\ln 2} 2^u + C = \frac{1}{\ln 2} 2^{\sqrt{y}} + C$$

19.) $\displaystyle\int \frac{1}{\sec\theta+\tan\theta}\,d\theta$

$$= \int \frac{\sec\theta-\tan\theta}{(\sec\theta+\tan\theta)(\sec\theta-\tan\theta)}\,d\theta$$

$$= \int \frac{\sec\theta-\tan\theta}{\sec^2\theta-\tan^2\theta}\,d\theta = \int \frac{\sec\theta-\tan\theta}{1}\,d\theta$$

$$= \ln|\sec\theta+\tan\theta| - \ln|\sec\theta| + c$$

21.) $\displaystyle\int \frac{4t^3-t^2+16t}{t^2+4}\,dt$

$$\begin{array}{r} 4t-1 \\ t^2+4 \,\big)\, \overline{4t^3-t^2+16t} \\ -(4t^3+16t) \\ \hline -t^2 \\ -(-t^2-4) \\ \hline 4 \end{array}$$

$$= \int \left[4t-1+\frac{4}{t^2+2^2}\right]dt$$

$$= 2t^2-t$$

$$+4\cdot\frac{1}{2}\arctan\left(\frac{t}{2}\right)+c$$

23.) $\displaystyle\int_0^{\frac{\pi}{2}} \sqrt{1-\cos\theta}\,d\theta = \int_0^{\frac{\pi}{2}} \sqrt{\frac{(1-\cos\theta)(1+\cos\theta)}{(1+\cos\theta)}}\,d\theta$

$$= \int_0^{\frac{\pi}{2}} \frac{\sqrt{1-\cos^2\theta}}{\sqrt{1+\cos\theta}}\,d\theta = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin^2\theta}}{\sqrt{1+\cos\theta}}\,d\theta$$

$$= \int_0^{\frac{\pi}{2}} \frac{|\sin\theta|}{\sqrt{1+\cos\theta}}\,d\theta = \int_0^{\frac{\pi}{2}} \frac{\sin\theta}{\sqrt{1+\cos\theta}}\,d\theta$$

$$= -2(1+\cos\theta)^{1/2}\Big|_0^{\frac{\pi}{2}}$$

$$= -2(1+\cancelto{0}{\cos\tfrac{\pi}{2}})^{1/2} - -2(1+\cancelto{1}{\cos 0})^{1/2}$$

$= -2(1) + 2(2)^{1/2} = 2^{3/2} - 2$

25.) $\int \frac{1}{\sqrt{e^{2Y}-1}}\,dy = \int \frac{e^Y}{e^Y\sqrt{(e^Y)^2-1}}\,dY$

(Let $u = e^Y \rightarrow du = e^Y dy$)

$= \int \frac{1}{u\sqrt{u^2-1}}\,dy = \text{arcsec}\, u + C$

$= \text{arcsec}\,(e^Y) + C$

26.) $\int \frac{6}{\sqrt{Y}(1+Y)}\,dy = 6\int \frac{1}{\sqrt{Y}(1+(\sqrt{Y})^2)}\,dy$

(Let $u = \sqrt{Y} \rightarrow du = \frac{1}{2}Y^{-1/2}dy = \frac{1}{2\sqrt{Y}}\,dy$

$\rightarrow 2\,du = \frac{1}{\sqrt{Y}}\,dy$)

$= 12\int \frac{1}{1+u^2}\,du = 12\arctan u + C$

$= 12\arctan\sqrt{Y} + C$

29.) $\int(\csc x - \sec x)(\sin x + \cos x)\,dx$

$= \int\left(\frac{1}{\sin x} - \frac{1}{\cos x}\right)(\sin x + \cos x)\,dx$

$= \int(\cancel{1} + \cot x - \tan x - \cancel{1})\,dy$

$= \ln|\sin x| - \ln|\sec x| + C$

34.) $\int e^{z+e^z}\,dz = \int e^z e^{e^z}\,dz$

(Let $u = e^z \rightarrow du = e^z\,dz$)

$$= \int e^u \, du = e^u + c = e^{e^z} + c$$

38.) $\displaystyle \int \frac{1}{\cos\theta - 1} \, d\theta = \int \frac{\cos\theta + 1}{(\cos\theta - 1)(\cos\theta + 1)} \, d\theta$

$$= \int \frac{\cos\theta + 1}{\cos^2\theta - 1} \, d\theta = \int \frac{\cos\theta + 1}{-\sin^2\theta} \, d\theta$$

$$= -\int \left( \frac{1}{\sin\theta} \cdot \frac{\cos\theta}{\sin\theta} + \frac{1}{\sin^2\theta} \right) d\theta$$

$$= -\int (\csc\theta \cot\theta + \csc^2\theta) \, d\theta$$

$$= -(-\csc\theta + -\cot\theta) + c$$

$$= \csc\theta + \cot\theta + c$$

39.) $\displaystyle \int \frac{1}{1+e^x} \, dx = \int \frac{1}{1+e^x} \cdot \frac{e^{-x}}{e^{-x}} \, dx$

$$= \int \frac{e^{-x}}{e^{-x} + e^0} \, dx = \int \frac{e^{-x}}{e^{-x} + 1} \, dx$$

$$= -\ln |e^{-x} + 1| + c$$

40.) $\displaystyle \int \frac{\sqrt{x}}{1 + x^3} \, dx = \int \frac{x^{1/2}}{1 + (x^{3/2})^2} \, dx$

(Let $u = x^{3/2} \rightarrow du = \frac{3}{2} x^{1/2} \, dx$

$\rightarrow \frac{2}{3} du = x^{1/2} \, dx$)

$$= \frac{2}{3} \int \frac{1}{1+u^2} \, du = \frac{2}{3} \arctan u + c$$

$$= \frac{2}{3} \arctan x^{3/2} + c$$

43.) $Y = \ln(\cos x)$, $0 \le x \le \frac{\pi}{3}$

$$\text{Arc} = \int_0^{\frac{\pi}{3}} \sqrt{1+(Y')^2}\, dx$$

$$= \int_0^{\frac{\pi}{3}} \sqrt{1+\left[\frac{1}{\cos x}\cdot(-\sin x)\right]^2}\, dx$$

$$= \int_0^{\frac{\pi}{3}} \sqrt{\frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x}}\, dx = \int_0^{\frac{\pi}{3}} \sqrt{\frac{1}{\cos^2 x}}\, dx$$

$$= \int_0^{\frac{\pi}{3}} \frac{1}{|\cos x|}\, dx = \int_0^{\frac{\pi}{3}} \frac{1}{\cos x}\, dx = \int_0^{\frac{\pi}{3}} \sec x\, dx$$

$$= \ln|\sec x + \tan x| \Big|_0^{\frac{\pi}{3}}$$

$$= \ln\left|\sec\frac{\pi}{3} + \tan\frac{\pi}{3}\right| - \ln|\sec 0 + \tan 0|$$

$$= \ln|2+\sqrt{3}| - \ln|\cancel{1}^{\,0} + 0|$$

$$= \ln|2+\sqrt{3}|$$

47.) $\int (1+3x^3)e^{x^3}\, dx$

$$= \int \left((1)e^{x^3} + (x)\cdot 3x^2 e^{x^3}\right) dx$$

$$= \int \left[D(x)\cdot e^{x^3} + (x)\cdot D(e^{x^3})\right] dx$$

$$= \int D(x\cdot e^{x^3})\, dx = xe^{x^3} + c$$

48.) $\int \frac{1}{1+\sin^2 x}\, dx = \int \frac{1}{1+\sin^2 x} \cdot \frac{1/\cos^2 x}{1/\cos^2 x}\, dx$

$$= \int \frac{\sec^2 x}{\sec^2 x + \tan^2 x}\,dx = \int \frac{\sec^2 x}{(1+\tan^2 x)+\tan^2 x}\,dx$$

$$= \int \frac{\sec^2 x}{1+2\tan^2 x}\,dx \quad \begin{pmatrix}\text{Let } u = \tan x \rightarrow \\ du = \sec^2 x\,dx\end{pmatrix}$$

$$= \int \frac{1}{1+2u^2}\,du = \int \frac{1}{1+(\sqrt{2}\,u)^2}\,du$$

$$= \frac{1}{\sqrt{2}} \arctan(\sqrt{2}\,u) + C$$

$$= \frac{1}{\sqrt{2}} \arctan(\sqrt{2}\tan x) + C$$

49.) $\int x^7 (x^4+1)^{1/2}\,dx = \int x^3 \cdot x^4 (x^4+1)^{1/2}\,dx$

(Let $u = x^4+1 \rightarrow du = 4x^3\,dx \rightarrow \frac{1}{4}\,du = x^3\,dx$ and $x^4 = u-1$ )

$$= \frac{1}{4}\int (u-1)u^{1/2}\,du = \frac{1}{4}\int (u^{3/2} - u^{1/2})\,du$$

$$= \frac{1}{4}\left(\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right) + C$$

$$= \frac{1}{10}(x^4+1)^{5/2} - \frac{1}{6}(x^4+1)^{3/2} + C$$