

Name: _____

Student ID# and Section: _____

Midterm Exam 1

Wednesday, February 3
MAT 21B, Temple, Winter 2021

Print names and ID's clearly. Write the solutions clearly and legibly. Do not write near the edge of the paper. Show your work on every problem. Be organized and use notation appropriately.

You are NOT allowed to consult the internet, Piazza, your classmates, friends or family members, tutors, or any other outside sources, etc. during the exam. You may NOT provide or receive any assistance from another student taking this exam. You may NOT use any electronic devices to look up hints or solutions for this exam. Show all work. Correct answers with insufficient or incorrect justification will receive little or no credit. By signing above you acknowledge that you have read and will abide by these conditions. Your exam will not be graded without your signature.

Signature: _____

Problem	Your Score	Maximum Score
1		20
2		20
3		20
4		20
5		20
Total		100

Problem #1 (20pts): Let $y = f(x)$ be a continuous function for $a \leq x \leq b$.

(a) State the Area Problem and define the Riemann integral over $[a, b]$ precisely as the limit of a Riemann Sum. (Draw the correct picture, and define and label grid points correctly.)

(b) State the Fundamental Theorem of Calculus, and explain in your own words why it is so important.

Problem #2 (20pts): Recall that $F(x) = \int_a^x f(t)dt$ denotes the area under the graph of f between a and x . Assuming f is a continuous function, draw this area in a diagram, and prove that $F'(x) = f(x)$.

Problem #3 (20pts): Use the substitution method to evaluate the following anti-derivatives: (Show every step.)

(a) $\int x^2 \sin x^3 dx$.

(b) $\int (3x - 1)^5 dx$.

(c) ~~$\int_1^2 F(\sin(2x)) \sin(x) \cos(x) dx.$~~

~~(Here F can be any continuously differentiable function.)~~

Delete this Problem!

Problem #4 (20pts): Let A denote the region between the graphs of $y = x^2$ and $y = x^4$ for $0 \leq x \leq 1$.

(a) Draw the region A and evaluate its area.

(b) Find the volume of the region obtained by rotating A about the x -axis, based on an integral with respect to x .

(Draw a picture and state which method you use, disk, washer or shell, and argue starting with a Riemann sum.)

(c) Find the volume of the region obtained by rotating A about the y -axis, based on an integral with respect to x .

(State which method you use, disk, washer or shell, and argue starting with a Riemann sum.)

Problem #5 (20pts): Consider the curve defined by the graph of the function $y = f(x) = \sin(x^2)$ between $x = 1$ and $x = 2$.

(a) Derive an integral formula for the arclength of the curve, starting with a Riemann sum. You need not evaluate the integral. (Draw a picture.)

(b) Derive an integral formula for the area of the region obtained by rotating the curve about the x -axis, starting with a Riemann sum. You need not evaluate the resulting integral. (Draw a picture.)