HOMEWORK QUIZ #2
Math 21b, Temple, Spring–05

Make sure to print your name and section number clearly, and sign your name below this, on the upper right-hand corner of this exam.

(1) Letting capital letters denote independent constants, what is the correct form of the partial-fraction representation of

$$\frac{x^3+2}{(x-1)^3(x^2+2x+3)^2}$$

(a) $\frac{A}{x-1}+\frac{B}{x^2+2x+3}$
(b) $\frac{A}{x-1}+\frac{Bx+C}{(x-1)^2}+\frac{Dx+E}{(x^2+2x+3)}+\frac{(Fx+G)^2}{(x^2+2x+3)^2}$
(c) $\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{(x-1)^3}+\frac{Dx+E}{(x^2+2x+3)}+\frac{Fx+G}{(x^2+2x+3)^2}$—CORRECT
(d) $\frac{A}{(x-1)^3}+\frac{B}{(x^2+2x+3)^2}$
(e) $\frac{A}{x-1}+\frac{A}{(x-1)^2}+\frac{A}{(x-1)^3}+\frac{Dx+E}{(x^2+2x+3)}+\frac{Dx+E}{(x^2+2x+3)^2}$
(f) None of the above

(2) Evaluate: $\int \frac{dx}{x^2+x+1}$. (Show each step!)

Solution: Step 1: Complete the square: $\int \frac{dx}{x^2+x+1}=\int \frac{dx}{(x+1/2)^2-1/4+1}=\int \frac{dx}{(x+1/2)^2+3/4}=\frac{1}{(3/4)}\int \frac{dx}{\left\{(\frac{x+1/2}{\sqrt{3}/2})^2+1\right\}}$
Step 2: Use substitution $u=\frac{x+1/2}{\sqrt{3}/2}$, $du=\frac{dx}{\sqrt{3}/2}$ to get

$$\frac{4}{3}\frac{\sqrt{3}}{2}\int\frac{du}{u^2+1}=\frac{2\sqrt{3}}{3}\tan^{-1}(u)+const=\frac{2\sqrt{3}}{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+const$$