HOMEWORK QUIZ #3
Math 21B, Temple, Spring–05

Print your name, section number and put your signature on the upper right-hand corner . Write only on the exam.

(1) Let **R** be the region bounded by $y = x + x^3$, $x = 0$, and $x = 1$. Use the shell method to find the volume of the solid produced by revolving **R** about the y-axis.

Solution: Partition $[0, 1]$ by $0 = x_0 < x_1 < \cdots < x_n = 1$ with $\Delta x = 1/n$ and $c_i = x_i$. Then the volume of the shell ΔV_i obtained by revolving the $i'th$ rectangle around the y-axis is $\Delta V_i = 2\pi x_i(x_i + x_i^3)\Delta x$, so the total volume V satisfies

$$V \approx \sum_{i=1}^{n} 2\pi x_i(x_i+x_i^3)\Delta x \to \int_0^1 2\pi(x^2+x^4)dx = 2\pi\left[\frac{x^3}{3} + \frac{x^5}{5}\right]_o^1 = \frac{16\pi}{15},$$

which gives the exact volume V in the limit $\Delta x \to 0$.

(2) Evaluate $\int_0^1 \sqrt{1-x^2}dx$.

Solution: Let $x = sin\theta$, $dx = cos\theta\ d\theta$, where for $0 \le x \le 1$ we only need $0 \le \theta \le \frac{\pi}{2}$. Substitution yields

$$\begin{aligned}\int_0^1 \sqrt{1-x^2}dx &= \int_0^{\pi/2} \sqrt{1-sin^2\theta}\ cos\theta\ d\theta = \int_0^{\pi/2} cos^2\theta\ d\theta \\ &= \int_0^{\pi/2} \frac{1}{2}\{1-cos2\theta\}\, d\theta = \left[\frac{\theta}{2} - \frac{sin2\theta}{4}\right]_{\theta=0}^{\theta=\pi/2} = \frac{\pi}{4}\end{aligned}$$