HOMEWORK QUIZ #4
Math 21B, Temple, Spring–05

Print your name, section number and put your signature on the upper right-hand corner . Write only on the exam.

(1) Let **R** be the region between the curves $y = x^2$ and $y = x^3$, and assume that **R** is a metal plate of constant density $\sigma = 1$. (a) Find the x-component of the center of mass, $\bar{x} = \frac{\int xc(x)dx}{Area}$. (b) Find the y-component of the center of mass, $\bar{y} = \frac{\int y\ c(y)\ dy}{Area}$. (You may use the formula $\int_c^d y\ c(y)\ dy = \int_a^b \frac{1}{2}\left\{f(x)^2 - g(x)^2\right\} dx$ when $\mathcal{R}$ is the region between the graphs of $y = f(x)$ and $y = g(x)$.)

Solution (a): The area $A = \int_0^1 x^2 - x^3\ dx = \frac{x^3}{3} - \frac{x^4}{4}]_0^1 = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$. Also, $\int xc(x)dx = \int_0^1 x(x^3 - x^4)\ dx = \frac{x^4}{4} - \frac{x^5}{5}]_0^1 = \frac{1}{4} - \frac{1}{5} = \frac{1}{20}$. Thus, $\bar{x} = \frac{12}{20} = \frac{3}{5}$.

Solution (b): Use $\int yc(y)dy = \int_0^1 \frac{1}{2}\left\{(x^2)^2 - (x^3)^2\right\}\ dx = \frac{1}{2}\left[\frac{x^5}{5} - \frac{x^7}{7}\right]_0^1 = \frac{1}{2}\left[\frac{1}{5} - \frac{1}{7}\right] = \frac{1}{35}$. Thus, $\bar{y} = \frac{12}{35}$.

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(2) Determine whether $I = \int_0^\infty \frac{e^{-x}\sin x}{1+x^2} dx$ converges or diverges. (Use theorems to justify each step.)

Solution: First by Theorem 3, the integral converges if it converges absolutely, that is, so long as $I = \int_0^\infty \left|\frac{e^{-x}\sin x}{1+x^2}\right| dx$ converges. But, $\left|\frac{e^{-x}\sin x}{1+x^2}\right| \leq e^{-x}$ because $\sin x \leq 1$ and $1 + x^2 \geq 1$ on the interval of integration $0 \leq x < \infty$. Since e^{-x} is integrable, it follows that I is integrable by the comparison test.